



Ore about Zero

Dr. Arun Kumar Mishra¹

¹Head, Department of Mathematics, P.U.P. College, Motihari- 845401 (B.R.A. Bihar University, Muzaffarpur) (Bihar)

Abstract

The origin of sexagesimal system may be related to early men's calculation of 360 days of a year. To quote from Florian Cajori's book, "A History of Mathematics" , "Cantor offers the following theory : At first the Babylonians reckoned the year at 360 days. This led to the division of the circle in 360 degrees, each degree representing the daily amount of the supposed yearly revolution of the sun around the earth. Now they were, very probably, familiar with the fact that the radius can be applied to its circumference as a chord 6- times, and that each of these chords subtends an arc measuring exactly 60 degrees. Fixing their attention upon these degrees, the division into 30-parts may have suggested itself to them. Thus, when greater precision necessitated a sub-division of the degree, it was partitioned into 60 minutes. In this way the sexagesimal system came up with the base 60 - system. There is also an alternate suggestion. It is easy to count up to 60 with 5 - fingers of one hand and 12 knuckles of the other. There was one more compelling reason. Sumerians did not have the radix - point, the symbol we use to separate the integer-part from the fractional-part of the number. Sixty is a super composite numbers; which is divided by -

1,2,3,4,5,6,10,12,15,20,30 and 60.

Use of fraction is minimised in the sexagesimal or base - 60 system. Over the years, the Babylonians developed a method whereby a vacant - space is left in between two numbers to indicate a place without value. While some European Historians likened the "vacant-space" with modern zero, but truly, it is noting like zero. The vacant space is always between two numbers, never at the end of a number, the way we designate 30 to 300. In this paper, I have tried to explain the concept of zero in the light of modern - findings.

Introduction

The Sumerian knowledge of Mathematics passed on the other ancient civilisations such as Egypt, India, China and othes. Egyptians and Indians used the decimal ot base 10 numeric system, the system which is prevalent today. Possibly, Egyptians invented the base 10 system for case of counting up to 10 using 10 fingers of the two hands, The Greeks, who contributed vastly in 'Geometry' and 'Number Theory' adopted a system based on the additive - principle and had no

need to introduce zero. They did not have separate symbols for numbers; rather they were drawn from their list of alphabets. They had separate symbols for higher numbers, namely-

$$100 = \lambda, 500 = \phi, \text{ etc.}$$

The highest number they could write was $10000 = u$, called "Myrad".

The Roman number system is similar to that of Greeks. Seven important symbols of the Roman number system are :-

I	=	1,
V	=	5,
X	=	10,
L	=	50,
C	=	100,
D	=	500,
& M	=	1000,

These seven symbols are used to construct all numbers, In this system a letter placed after another of greater value adds and a letter placed before another of greater value subtracts. Thus -

III	=	3,
IX	=	9,
XI	=	11,
MMXVI	=	2016

We can easily imagine the grief of a Roman mathematician, if he has to write a large number. For example, to write 100 thousands, he has to write, M one after another, one hundred times.

Zero at a Glance

An eminent fact about zero is that zero is the contribution of India to develop Mathematics. Laplace (1749-1827) was one of the greatest mathematician of France. Laplace had written about zero, "It is India that gave us the ingenious method of expressing all numbers by means of ten symbols, each symbol receiving a value of position as-well-as an absolute-value; a profound and important idea, which appears so simple to us now that we ignore its true & merit. But it's very simplicity and the great ease which it has lent to computations put our arithmetic in the first rank of useful inventions; and we shall appreciate the grandeur of the achievement the more when we remember that it escaped the genius of Archimedes and Apollonius, two of the greatest men produced by antiquity." Traditionally, Indians used the Brahmi numerical system, Brahmi numerical system was a decimal system but did not have a zero. It was also not a place-value system and separate symbols were used for 20,30,40,50 and others. Aryabhata was most important post - Vedic mathematician and astronomer, lived during 476-550 CE. Aryabhata did

not use zero. Possibly, under the influence of the Greeks, he introduced a numerical system based on the "Sanskrit-Alphabets". Like the Greeks, Aryabhata drew his numbers from the Sanskrit alphabets. In Sanskrit, the first twenty five letters are called "Varga Letters" and the rest are "Avarga Letters". Aryabhata assigned value 1,2,3,.....,25 to the 25 Varga Letters. The avarga letters were assigned values 30,40,50,60,70,80,90 & 100 respectively. Aryabhata also assigned numeric values to the nine Sanskrit vowels, He then enunciated the rules to write down the numbers. If a consonant is in conjunction with a vowel, then the consonant value has to be multiplied by the place-value of the vowel. Aryabhata was essentially an astronomer and so he needed large numbers. By his assignment of large numeric values to the Sanskrit vowels, he ensured that he could write very large numbers.

Brahmagupta and Zero (Sunya)

It was Brahmagupta (598-C-670C.E.) in India. who invented zero at the age of 30 years. Brahmagupta wrote his magnum opus, "Brahmasphutasidhanta". "Brahmasphutasidhanta" was the first book, which mentions zero as a number under the name "sunya"; which is still used in India to indicate nothingness, vacuum. Brahmagupta was born in 520 saka (655 Vikrami or 589 A.D.) in the regin of King Vyaghramukha, belonging to the capa family. At the age of 30, he wrote in 550 saka (628 A.D.) his well-known treatise on Astronomy known as the "Brahmasphutasidhanta" which is corroborated by the statement in the Vishaudharmottara Purana, a chapter on the Brahma-sidhanta. His other treatise, entitled the Khandna-Khandyaka, which is a karana book, was completed in 587 saka (665 A.D.) According to some authors, Brahmagupta was the grandson of Visnugupta, and the family-suffix (Gupta) indicated that he belonged to the. Vaisya" family. Brahmagupta was in the service of the "King of Rewah", known as 'Vyagrabhata'. Brahmagupta was a great critic. He did not spare any of his predecessors, like Aryabhata, Varahmihira, Srisena, Visnucandra and others. Later on his influence on the writing of the succeeding generations has been immense. Of course, Brahmagupta defined zero as the result of subtraciting a number from itself. He also gave the rules of mathematical operations with zero. If 'a' stands for a number, then according to Brahmagupta,

$$(i) \quad a+0 = a$$

$$(ii) \quad a-0 = a$$

$$(iii) \quad ax0 = a$$

$$(iv) \quad \frac{a}{0} = \frac{a}{0}$$

$$(v) \quad \frac{0}{0} = 0$$

His rules of addition, subtraction & multiplication with zero are valid still today. However, his rules for division by zero were wrong, He was non committal about $\frac{a}{0}$ and he thought. $\frac{0}{0} = 0$

Now these days, we understand that division by zero is not a mathematically defined operation, as there is no number which when multiplied by 0 gives x (assuming $x \neq 0$). Since any number multiplied by zero is zero, so the expression $\frac{0}{0}$ also has no defined value.

Bhaskara II and Zero

Approximately five hundred years after Brahmagupta, the correct rules for division by zero were given by the Indian Mathematician, Bhaskara-II; who is also known as Bhaskaracharya (1114-1185 C.E.) in his mathematical treatise, "Lilavati", named after his daughter by assuming 'a' as a non-zero number as -

$$a \div 0 = a$$

$$0^2 = 0$$

$$\sqrt{0} = 0$$

$$0^3 = 0$$

$$\sqrt[3]{0} = 0$$

$$\frac{a}{0} = \infty$$

$$ax0 = 0$$

$$\frac{ax0}{0} = a$$

whose denominator is zero. He did not call it "infinity", rather he called it "Khahara." He correctly understood its nature and in "Lilavati", wrote,

"There is no change in "Khahara" figure if something is added to or subtracted from the same."

The Arabian Traders and Zero

The Indian Zero (Sunya) traveled from India through the Arabian traders during 800-1500 C.E. Arabic scholars made spectacular progress in Mathematics and Astronomy. Caliph Harun - al-Rshid (reigned 786-809 C.E.) established an academy or intellectual centre at Bagdad called "Bayt al - Hikmah" or "House of Wisdom". Under his patronage, the pursuit of knowledge became a dominant feature of the caliphate and scholars were encouraged to translate medieval works and the vast Greek and Indian literature pertaining to Philosophy, Mathematics, Natural Science, and Meicine were translated, into the Arabic language. Renowed Arabic mathematician, Al-khwarizmi (ca-780-ca850) studied in the 'House of Wisdom' and later become its director. From his famous book, "Kitab-al- jabr wa I-muquabhala", the modern world got the term "Algebra",. The court of Bagdad received a gift of Brahmagupta's book and Al khwarizmi oversaw its translation. He understood the importance of zero as a number and place holder. He did not call it. 'zero' or 'sunya', rather he called it "sifr". In Arabic, 'sifr' means empty. Synthesizing the Arabian and Indian knowledge, in 825 C.E., Al-khwarizmi published a book entitled, " On the calculation with Hindu Numerals".

Conclusion

Zero entered Europe in the 12th century when Al khwarizmi's book was translated into Latin under the little, " al-Khwarizoni into Latin under the little, " al- Khwarizmi on the Numerals of the Indians". Italian mathematician, Leonardo Fibonacci (ca. 1170-1250 CE), also called Leonardo

of Pisa, popularised the use of zero in Europe. In 1202, Fibonacci wrote a book, "Liber abaci" (Book of calculation). The book was instrumental in spreading the use of zero in Europe. The book begins with,

"The nine Indian figures are:

9 8 7 6 5 4 3 2 1

With these nine figures and with sign 'O' which the Arabs call zephir any number whatsoever is written, as is demonstrated below."

The book, "Liber Abaci" was hugely popular in Europe and the second edition appeared in the year, 1228. Slowly, the Europeans accepted zero as a number and placeholder, and Italian "Zephir" contracted to the present form "Zero".

References

Aaboe, Asgar; Episodes from the Early History of Mathematics; Random House, 1964.

Bell E.T.; Men of Mathematics; Mc Graw Hill, 1945.

Cajori, Florian; A History of Mathematical Notations; Open curt.
Chicago, 1928.

Hardy, G.H.; A Mathematician's Apology; Cambridge University Press, London, 1967.

Whitehead, Altrlead N., Science and the Modern World; Cambridge University Press, 1927.