International Journal of Transformation in Applied Mathematics & Statistics
Vol. 1, Issue 1 - 2018

SOME SIMPLE OBSERVATION ON GENERALISED LATTICE

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ABSTRACT

This paper is a study about somesimple observation on generalised lattice.

KEYWORDS: Graded Poset, Directed Set, Generalisedlattice.

INTRODUCTION

In this paper, we study some simple observation on generalised lattice. Swamy and murty [2] introduced the concept of a generalised lattice and elaborately studied by Kishore and murty [3]

Let A be a finite subset of P.

Define

L (A) = { x∈ P / x≤a for all a∈ A} and

U (A) = { x∈ P / a≤x for all a∈ A} then

L (P) = {L(A) / A is a finite subset of P} and

U (P) = {U(A) / A is a finite subset of P} are semilattices under set inclusion.

GENERALISED MEET SEMILATTICE [2]

P is said to be a generalised meet lattice if for every non empty finite subset A of P, there exists a non-empty finite subset B of P such that, $x \in L(A)$ if and only if $x \le b$ for some $b \in B$.

It is observed that if P is a generalised meet (join) semilattice, then for any $L(A) \in L(P)$ $[U(A) \in U(P)]$ there exists a unique finite subset B of P such that $L(A) = \bigcup_{b \in B} L(P)$

$[U(A) = \bigcup_{b \in B} U(P)]$ and the elements of B are mutually incomparable.

GENERALISED LATTICE [2]

P is said to be a generalised lattice if it is both generalised meet and join semilattice.

REMARK [3]

Clearly every finite poset which is directed below (above) is a generalised meet (join) semilattice and every meet (join) semilattice is a generalised meet (join) semilattice.

In general every lattice as well as every finite directed poset is a generalised lattice. The posets in figures: 1 and 2 are generalised lattices which are not lattices.

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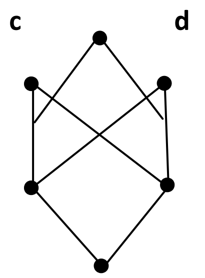

Figure 1

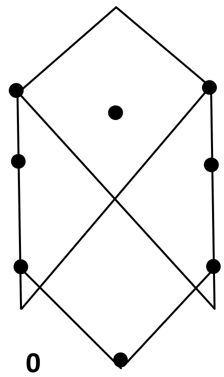

Figure 2

THEOREM: 1.1[2]

A generalised lattice is a lattice if and only if it is either a meet semilattice or a join semilattice.

DIRECTED SET [4]

A poset P is called a directed set if for any a, b $\in$ P there exists x$\in$ P such that a$\leq$x and b$\leq$x.

GRADED POSET [4]

A poset P is graded if all maximal chains in P have the same length.

REMARK [4]

Any directed poset be a graded but converse need not be true.

THEOREM: 1.2

Every finite graded poset be a generalised lattice.

Proof:

By the remark 1.5, obviously the statement is true.

THEOREM: 1.3

Any generalised lattice need not be graded.

PROOF

For example see Fig [2] is a generalised lattice but intervals [0, c] and [0, d] are all maximal chains have not the same length; therefore any generalised lattice need not be graded.

REFERENCES

[1]. Gratzer. G., General Lattice Theory, Birkhausar Verlag Basel, 1978.

[2]. M.K.Murty, U.M. Swamy., Distributive partially ordered sets, The Aligarh Bull. of Math.Vol.8, 1978.

[3]. P.R.K.Kishore, M.K. Murty, V.Sri ramani and M.D.P. Padnaik., On generalised lattices, South East Asian Bull. Math. 33(5) (2009), 1091-1104.

[4]. Santhi. V.K. Topics in Commutative Algebra, PhD thesis, Madurai Kamaraj University, 1992.

