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Euclidean Geometry in the Mirror of History

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Abstract

Geometry is an ornamental surrounding of mathematical science. It explains the position or place of our considering unit in our life. In a clear-cut way, one can say that the formal way of geometry started with the time of Euclid (….. around 300 B.C.), who compiled and systematized all of what was known about plane and solid geometry in his period. Euclid's 'Element' was so solid that it was virtually the 'Bible of Geometry' for over two thousand years-one of the most enduring works of all time. Everything was in favour with Euclid. The reason for its long-existence is due to the best logic in step-by-step explanation. I have tried a briet account for its long and meaningful journey.

Introduction

The principal reason for the long existence of 'Elements' of Euclid was that Euclid was the founder of rigor in mathematics ' The Elements' began with very simple concepts, definitions and so forth, and gradually built up vast body of results organised in such a way that any given result depended only on foregoing results. Thus, there was a definite plan to the work, an architecture which made it strong and sturdy. In Euclid's Elements' the stuff out of which proofs were constructed was human-language-that elusive, tricky medium of communication with so many hidden pitfalls. Every word has its own meaning. The more common the word, the more associations we have with it, and the more deeply rooted is its meaning. Therefore, when someone gives a definition for a common-word in the hopes that we will abide by that definition, it is a foregone conclusion that we will not do so but will instead be guided, largely unconsciously, by what our minds find in their associative stores. Euclid's Elements dealt with points, angles, planes, triangles, straight lines, circles and so forth. Euclid had accepted the most general practical words. He felt that the points and lines of his 'Elements' were indeed the points and lines of the real world. So by not making sure that all associations were dispelled, Euclid was inviting to let their powers of association run free.

This sounds almost anarchic, and is a little unfair to Euclid. He did set down axioms (or postulates), which were supposed to be used in the proofs of Propositions. In fact, nothing than those axioms and postulates was supposed to be used. But this is where he slipped up, for an inevitable consequence of his using ordinary words was that some of the images conjured up by those words crept into the proofs, which he performed. There is no 'jump' in the 'Elements' of Euclid. Euclid was a penetrating thinker, who had not any simple minded errors. Nonetheless, gaps are there, creating slight imperfections in a classic work. But this is not to be complained about. One should merely gain an appreciation for the difference between absolute rigor and relativistic rigor. In the long run, Euclid's lack of absolute rigor was the cause of some of the most fertile path breaking in mathematics, over two thousand years after completion of 'Elements'.

Ground Story

Euclid gave five postulates to be used as the "ground story" of the infinite skyscraper of geometry, of the which his 'Elements' completed only the first several hundred stories. The first four postulates are rather terse and elegant. These are.

- Postulate-1: A straight line segment can be drawn joining any two points.
- Postolate-2: Any straight line segment can be extended indefinitely in a straight line.
- Postolate-3: Given any straight line segment, a circle can be drawn having the segment as radius and one end- point as center.
- Postolate-4: All right angles are congruent.
- Postolate-5: If two lines are drawn which intersect a third in such a way that sum of the inner angles on one side is less than two right angles, then the two lines inevitably must interest each other on that side it extended far enough.

Euclid considered the fifth postulate to be somehow inferior to the first four, since he managed to avoid using it in the proofs of the first twenty eight propositions. Thus, first twenty eight propositions belong to what might he called " four postulate geometry, " that part of geometry which can be derived on the basis of the first four postulates of the 'Elements' without the help of the fifth postulate. It is also often called 'Absolute Geometry'. The fifth postulate did not share their grace. Euclid would have found it far preferable to prove this ugly duckling, rather than to have to assume it , but he found no proof, and so he adopted it.

But the disciples of Euclid were no happier about having to assume the fifth postulate. Over the centuries, untold numbers of people gave untold year of their lives in attempting to prove that fifth postulate was itself a part of four postulate geometry. By 1763, at least twenty eight different proofs had been published –all erroneous! They were all criticized in the dissertation of one G. S. Kluge. All of these erroneous proofs involved a confusion between everyday intuition and strictly formal properties. It is safe to say that today, hardly any of these "proofs" holds any mathematical or historical interest- but there are certain exceptions!

'Changed Faces of the Elements'

Girolamo Saccheri (1667-1733) had the ambition to free Euclid of every flaw. Based on some earlier work he had done in logic. Saccheri decided to try a novel approach to the proof of the fifth postulate of Euclid. Suppose we assume its opposite, then work with that as the fifth postulate……. Surely after while we shall have create a contradiction. Since, no mathematical system can support a contradiction, so we shall have shown the unsoundness of the fith postulate.

Saccheri worked out proposition after proposition "Saccherian Geometry" and eventually became tired of it. At one point, he decided that he had reached a proposition which was "repugnant to the nature of the straight line" that was what he had been hoping for- to his mind, it was the long- sought contradiction. At that point, he published his work under the title, "Euclid Freed of Every Flaw", and then expired.

But in so doing, he robbed himself of much posthumous glory, since he had unwittingly discovered what "came later to be known as "hyperbolic geometry." Fifty years after Saccheri, J.H. Lambert repeated the "near miss", this time coming even closer, if possible. Finally, forty years after Lambert and ninty years after Saccheri, non- Euclidean geometry' was recognized for what it it was- an authentic new brand of geometry, a bifurcation in the hitherto single stream of mathematics. In 1823, non- Euclidean geometry was discovered simultaneously, in one of those inexplicable coincidences, by a Hungarian mathematician, Janos (or Johann) Bolyai at the age of twenty one years, and a Russian mathematician, Nikolay Lobachevskiy, at the age of thirty years. And ironically, in that same year, the great French mathematician, Adrien-Marie Legendre came up with what he was sure was a proof of Euclid's fifth postulate, very much along the lines of Saccheri.

Incidentally, Bolyai's father, Farkas (or Wolfgang) Bolyai, a close friend of the great Gauss, invested much effort in trying to prove the fifth postulate of Euclid's 'Elements.' In a letter to his son Janos, he tried to dissuade him from thinking about such matters: 'You must not attempt this approach to parallels. I know this way to its very end. I have traversed this bottomless night, which extinguished all light and joy of my life. I entreat you, leave the science of paralles alone………….

I thought I would sacrifice myself for the sake of the truth. I was ready to become a martyr who would remove the flaw from geometry and return it purified to mankind. I accomplished monstrous, enormous labors; my creations are for better than those of others and yet I have not achieved complete satisfaction. For here it is true that si-upallum a summ discessit, vergit ad imum. I turned back when I saw that no man can reach the bottom of this night. I turned back unconsoled, pitying myself and all mankind……… I have traveled past all reefs of this infernal Dead Sea and have always come back with broken mast and torn sail. The ruin of my disposition, and my fall date back to this time. I thought-lessly risked my life and happiness- a. Caesar out rihil.'

But later when he convinced his son really" had something", he urged him to publish it, anticipating correctly the simultaneity which is so frequent in scientific discovery:

"when the time is ripe for certain things, these things appear in different places in the manner of violets coming to light in early spring,"

How true this was in the case of non-Euclidean geometry! In Germany, Karl Friedrick Gauss himself and a few- others had more or less independently hit upon non-Euclidean ideas. These included a lawyer, F. K. Schweikart who sent a page describing a new "astral" geometry to K.F Gauss in 1818.It must be remembered also that Schweikart's nephew, F. A. Taurinus did non-

Euclidean geometry, and F. L. Wachter, a student of the great Gauss; who died in 1817 at the age of twenty five years, having found several deep and fruitful results in non- Euclidean geometry.

Conclusion

The clue to non-Euclidean geometry was "thinking straight" about the propositions which emerge in geometry like Saccheri's and Lambert's. The Sacherian's propositions are only "repugnant to the nature of the straight line", it we cannot free ourselves of pre-convinced notions of what "straight line" must mean. If however, we can divest ourselves of those pre-convinced images, and merely let a "straight line" be something which satisfies & the new propositions, then we have achieved a radically new-view point. One can let the meaning of "point," "line" and so on the determined by the set of theorems (or propositions) in which they occur. This was the great realization of the discoveries of non-Euclidean geometry. They found different sorts of non- Euclidean geometries by denying Euclid's fifth postulate in different ways and following out the consequences. In a real way, we can say that F. K. Schweikart, F. A. Taurinus, F. I. Wachter and Saccheri did not deny the fifth postulate directly, but rather, they denied an equivalent postulate, called the ' parallel postulate', which run as follows :

"Give any straight line, and a point not on it, there exists one, and only one, straight line which passes through that point and never intersects the first line, no matter how far they are extended." Of course, the second straight line then said to be 'parallel' to the first. If we assert that no such lines exist, then we reach at "elliptical geometry." If we assert that at least two such lines exist, we reach at "haperbolic geometry." Incidentally, the reasom that sach variations are still called "geometries" is that core- element absolute, or four- postulate geometry- is embedded in them. It is the presence of this minimal core which makes it sensible to think of them as describing properties of some sort of geometrical space, even if the space is not as intuitive as ordinary space.

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