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Magneto Hydrodynamics Turbulent dusty Flow with Rotational Symmetry

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Introduction

MHD Turbulence describes, turbulence in an electrically conducting, magnetized fluid (Biskam 2003). Strictly speaking MHD only applies to collision dominated fluids. However it is often a useful guide to the behaviour of magnetized plazmas even in the collisionless limit, turbulence is a generic property of large scale fluid flows. Hydrodynamic (HD) turbulence is a familiar phenomenon, flows of human dimensions commonly reach high Reynolds numbers; values in excess of 10^4 are achieved in the air we push aside when we walk and in the water we disturb when we swim. By contrast the limited electrical conductivity of available fluids makes it difficult to excite flows with high magnetic Reynolds numbers in terrestrial laboratories.

It was pointed out by Taylor (1953) that the equations of motion of ordinary turbulence are related with the pressure gradient and the acceleration of a fluid particle. Batchelor (1951) Jain (1962) and Kishore and Mishra (1970) obtained the expressions for pressure covariance and acceleration converiance in ordinary and MHD turbulence. Kishore and Singh (1984) discussed the effect of coriolis force on acceleration covariance in ordinary turbulence with rotational symmetry following Kishore and Dixit (1970). In this paper an attempt has been made to study the effect of dust particles on acceleration covariance in nearly isotropic, homogenous MHD turbulence. It is the extension of the work done by Dixit (1988).

Discussion of the Problem

The equations of motion and of continuity of an incompressible viscous fluid are those of Chandrasekhar (1955) : namely

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_k}(u_i u_k - h_i h_k) = -\frac{\partial w}{\partial x_i} + v\nabla^2 u_i + f(ui - vi) \quad .. \quad (1)$$

$$\frac{\partial h_i}{\partial t} + \frac{\partial}{a x_k}(h_i u_k - u_i h_k) = \lambda \nabla^2 h_i \quad .. \quad (2)$$

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{a x_j} = -\frac{K}{m_s}(U_i - V_i) \quad .. \quad (3a)$$

$$\frac{\partial h_i}{\partial x_i} = \frac{\partial u_i}{a x_i} = \frac{\partial v_i}{\partial x_i} = 0 \quad .. \quad (3b)$$

Where ms = $^4/_3\, \pi R\frac{R}{S}\rho_S$ is the mass of a single spherical dust particle of Radius R_S.

K = 6 $\pi R_S \rho_S$ by the Stoke's drag formula

F = Kn/ ρ has dimensions of frequency

N = Constant number density of dust particles

ρ_S = constant density of the material in dust particle

v_i (x, t) = dust velocity components.

u_i (x, t) = fluid velocity components.

Let $A_{\bar{i}}^{'}\,(x_{\bar{i}}^{'}, t^{'})$ denote the ith component of the acceleration of a fluid particle which is instantaneously at the point $P^{'}(x_i^{'})$ and at the time t′. Then

$$A_i(x_i^{'}, t^{'}) = \frac{Du_i^{'}}{Dt^{'}} =$$

$$= \frac{\partial u_i^{'}}{\partial t^{'}} + u_k^{'}\frac{\partial u_i^{'}}{\partial x_k^{'}} =$$

$$= \frac{\partial w'}{\partial x'_i} + \nu \nabla_{x'}^2 u_i^{'} + \frac{\partial}{\partial x'_k} h_i^{'} h_k^{'} + f\,(u_i^{'} - v_i^{'}) \quad .. \qquad (4)$$

Similarly, if $A_j^{''}(x_j^{''}\, t'')$ denote jth component of the acceleration of another fluid particle which is instantaneously at the point $P^{''}(x_j^{''})$ and at time t′′ we can write

$$A_j^{''}(x_j^{''}, t^{''}) = -\frac{\partial^{''}}{\partial x_j^{''}} + \nu \nabla_{x''}^2 u_j^{''} + \frac{\partial}{\partial x_i^{''}} h_j^{''} h_i^{''} + f(u_j^{''} - v_j^{''}) \quad .. \qquad (5)$$

Therefore,

$$A_i^{'} A_j^{''} = \frac{\partial w^{'} \partial w^{''}}{\partial x_i^{'} \partial x_j^{''}} - -\nu \nabla_{x''}^2 \frac{\partial}{\partial x_i^{'}} \overline{w^{'} u_j^{''}} - \nabla_{x'}^2 \frac{\partial}{\partial x_j^{''}} w^{''} u_i^{'}$$

$$- \frac{\partial w^{'}}{\partial x_i^{'}} \frac{\partial}{\partial x_l^{''}} h_j^{''} h_i^{''} \frac{\partial w^{''}}{\partial x_j^{''}} \frac{\partial}{\partial x_k^{'}} h_j^{'} h_k^{'} +$$

$$+ \nu \nabla_{x'}^2 u_i^{'} \frac{\partial}{\partial x_l^{''}} h_j^{''} h_l^{''} \nu \nabla_{x''}^2 u_j^{'} \frac{\partial}{\partial x_k^{'}} h_i^{'} h_k^{'} +$$

$$+ \nu \nabla_{x'}^2 \nabla_{x''}^2 u_i^{'} u_j^{''} + \frac{\partial}{\partial x_k^{'}} \frac{\partial}{\partial x_l^{'}} h_i^{'} h_k^{'} h_j^{''} h_l^{''} -$$

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$$-f\frac{\partial}{\partial x_i'}\left(w'u_j'' - w'v_j''\right) - \frac{\partial}{\partial x_j''}\left(w''u_j' - w''v_i'\right) +$$

$$+\nu f\,\nabla_{x'}^2\left(u_i'u_j'' - u_i'v_j'\right) + \nu f\,\nabla_{x''}^2\left(u_i'u_j'' - v_i'u_j''\right)$$

$$+f\frac{\partial}{\partial x_k'}\left(h_i'h_k'u_j'' - h_i'h_k'u_j''\right) + f\frac{\partial}{\partial x_l''}\left(u_i'h_j''h_l'' - v_i'h_j''u_l''\right)$$

$$+f^2 u_i'u_j'' - u_i'v_j'' - v_i'u_j'' + v_i'v_j'' \qquad \ldots \qquad (6)$$

Since the dust grains are taken as non conducting and therefore $\overline{h_i'u_j''} = \overline{h_j''v_j'}$ =0, we assume that the instantaneous velocities at one point remain unaffected by the dust particles of the other point $\overline{u_i'v_j''} = \overline{u_j''v_i'} = 0$.

Taking the average and using the conditions of homogeneity and rotational symmetry we have

$$\overline{A_i'A_j''} = \frac{\partial^2 \overline{w'w''}}{\partial\xi_i\partial\xi_j} + \frac{\partial^2}{\partial\xi_i\partial\xi_j}\overline{w'h_j''h_l''} + \frac{\partial^2}{\partial\xi_j\partial\xi_k}\overline{w''h_i'h_k'} +$$

$$+\nu\nabla^2\left[\frac{\partial}{\partial\xi_l}\overline{u_i'h_j''h_l''} + \frac{\partial}{\partial\xi_k}\overline{u_j'h_i'h_k'}\right] +$$

$$+\nu^2\nabla^4\overline{u_i'u_j''} - \frac{\partial^2}{\partial\xi_k\partial\xi_l}\overline{h_i'h_k'h_j''h_l''} + 2\nu f\,\nabla^2\overline{u_i'u_j''}$$

$$-f\frac{\partial}{\partial\xi_k}\left(\overline{h_i'h_k'u_j''} - \overline{h_i'h_k'v_j''}\right) + f\frac{\partial}{\partial\xi_l}\left(\overline{u_i'h_j''h_l''} - \overline{v_i'h_j''h_l''}\right)$$

$$+f^2\left(\overline{u_i'u_j''} - \overline{v_i'v_j''}\right) \qquad \ldots \qquad (7)$$

Where,

$$\xi_i = x_i'' - x_i', \quad \frac{\partial}{\partial\xi_i} = \frac{\partial}{\partial x_i''} = -\frac{\partial}{\partial x_i'}$$

$$\nabla_{x'}^2 = \nabla_{x''}^2 = \nabla^2$$

It follows from the properties of a normal joint distribution of h, given by Chandrasekhar (1955), that

$$\overline{h_i'h_k'h_j''h_l''} = \overline{h_i'h_j''h_k'h_l''} + \overline{h_i'h_l''h_k'h_j''} + \overline{h_i'h_k'h_j''h_l''}$$

$$H_{ik;ji} = H_{ij}H_{kl} + H_{il}H_{kj} + H_{ik}(O,O)H_{jl}(O,O) \qquad \ldots \qquad (8)$$

Therefore, using short notations

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$$\frac{\partial^2}{\partial \xi_k \ \partial \xi_l} H_{ik\,;jl} = \frac{\partial^2}{\partial \xi_k \ \partial \xi_l} H_{ij}\, H_{kl} + \frac{\partial^2}{\partial \xi_k \ \partial \xi_l} H_{il} H_{kj} +$$

$$+ \frac{\partial^2}{\partial \xi_k \ \partial \xi_l} H_{ik}\,(0,0) H_{jl}(0,0)$$

or

$$\frac{\partial^2}{\partial \xi_k \ \partial \xi_l} (H_{ik;ji}) = \frac{\partial}{\partial \xi_k} H_{il} \frac{\partial}{\partial \xi_l} H_{kj} \quad .. \qquad (9)$$

The first and the last terms vanish because H_{ik} (0,0), H_{jl} (0,0) are constants and is solenoidal in indices K and l H_{il} and H_{kj} being isotropic second order tensors, Solenoidal in their indices, can be represented in terms of a single defining Scalar H as

$$H_{il} = \frac{H'}{r}\xi_i\xi_l - (rH' + 2H)\,\delta_{il}$$

and

$$H_{kj} = \frac{H'}{r}\xi_k\xi_j - (rH' + 2H)\,\delta_{kj}$$

Where Primes denote differentiation with respect to r. Hence we have

$$\frac{\partial}{\partial \xi_k} H_{il} \frac{\partial}{\partial \xi_l} H_{kj} = \left[\frac{12H'^2}{r^2} + \frac{2H''}{r}\right]\xi_i\xi_j - [6H'2 - 2rH'H'']\delta_{ij}. \qquad (10)$$

Let us put, as in Jain (1962)

$$P(r) = \overline{w'w''}\ Q_{ij} = \overline{u_i' u_j''},\ Hij = \overline{h_i' h_j''}\pi_{jl} = \overline{w' h_i'' h_i''}$$

$$\pi_{jk} = \overline{w'' h_i' h_{k'}'}\ \ S_{ik,\,j} = \overline{h' h_k' u_j'},\ \ S_{jl,\,i} = \overline{u_i' h_j'' h_l''}$$

$$H_{ik,\,jl} = \overline{h_i' h_k' h_j'' h_l''},\ \ L_{ik,\,j} = \overline{h_i' h_k' v_j''},\ \ R_{ij} = \overline{v_i' v_j''} \quad \ldots \qquad (11)$$

If we substitute the expressions (9) and (11) in (7) the expressions for the acceleration covariance are

$$\overline{A_i' A_j''} = \frac{\partial^2 P(r)}{\partial \xi_i \partial \xi_j} + \frac{\partial^2}{\partial \xi_i \partial \xi_j}\pi_{jl} + \frac{\partial^2}{\partial \xi_j \partial \xi_k}\pi_{ik} +$$

$$\nu\nabla^2 \frac{\partial^2}{\partial \xi_l}\ (-S_{jl,\,i}) - \frac{\partial^2}{\partial \xi_k}\ S_{ik,\,j} + \nu^2\nabla^4\ Q_{ij} -$$

$$\frac{\partial^2}{\partial \xi_k \partial \xi_l}\ H_{ki,\,jl} + 2\nu f\nabla^2\ Q_{ij} - f\frac{\partial}{\partial \xi_k}(S_{ik,\,j} - L_{ik,\,j})$$

$$+ f \frac{\partial}{\partial \xi_l} [-S_{jl,i} + L_{jl,i}] + f^2 (Q_{ij} - R_{ij}) \quad .. \qquad (12)$$

Now,

$$\frac{\partial}{\partial \xi_l} S_{jl,i} = \frac{\partial}{\partial \xi_k} S_{ik,j} , \frac{\partial^2}{\partial \xi_l \partial \xi_l} \pi_{jl} = \frac{\partial^2}{\partial \xi_i \partial \xi_k} \pi_{ik}$$

So that equation (2.2.12) becomes.

$$\overline{A_i' A_j''} = \frac{\partial^2 P(r)}{\partial \xi_i \partial \xi_j} + 2 \frac{\partial^2}{\partial \xi_i \partial \xi_j} \pi_{jl} - 2\nu \nabla^2 \frac{\partial}{\partial \xi_k} S_{jk,i} +$$

$$+ \nu^2 \nabla^2 Q_{ij} - \frac{\partial}{\partial \xi_k} H_{il} \frac{\partial}{\partial \xi_l} H_{kj} + 2\nu \nabla^2 Q_{ij} -$$

$$- 2f \frac{\partial}{\partial \xi_k} S_{jk,i} + 2f \frac{\partial}{\partial \xi_k} L_{jk,i} + f^2 (Q_{ij} - R_{ij}) \quad .. \qquad (13)$$

We can express $\overline{A_i' A_j''}$ as

$$\overline{A_i' A_j''} = \alpha (r, t) \xi_i \xi_j + \beta(r, t) \delta_{ij} \qquad (14)$$

Where α, β are its defining scalars.

But

$$= \frac{\partial^2 P(r)}{\partial \xi_i \partial \xi_j} = \frac{\xi_i \xi_j}{r} \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial p(r)}{\partial r}\right) + \frac{1}{r} \frac{\partial p}{\partial r} \delta_{ij} \qquad (15)$$

or

$$= \frac{\partial^2 P(r)}{\partial \xi_i \partial \xi_j} = \frac{16 \xi_i \xi_j}{r^2} (Q'^2 + H'^2) \frac{1}{r} \frac{\partial p(r)}{\partial r} + \delta_{ij} \qquad (16)$$

and as in Jain (1962)

$$Q_{ij} = \frac{Q'}{r} \xi_i \xi_j - (rQ' + 2Q) \delta_{ij} \qquad (17)$$

$$\nabla^2 Q_{ij} = \left[\frac{Q''}{r} + \frac{4}{r^2} Q'' - \frac{4}{3} Q'\right] \xi_i \xi_j -$$

$$- \left[rQ''' + 6Q'' + \frac{4}{r} Q'\right] \qquad (18)$$

$$\nabla^4 Q_{ij} = \left[\frac{Q^{v}}{r} + \frac{8Q^{iv}}{r^2} - \frac{24}{r^4} Q'' + \frac{24}{r^5} Q'\right] \xi_i \xi_j -$$

$$- \left[rQ^{v} + 10Q^{iv} + \frac{16}{r} Q''' - \frac{8Q''}{r^2} + \frac{8}{r^3} Q'\right] \delta_{ij} \qquad (19)$$

Also π_{jl} , being isotropic, must be of the form

$$\pi_{jl} = \pi_1 \xi_i \xi_l + \pi_2 \delta_{ij}$$

and

$$\frac{\partial^2}{\partial \xi_i \partial \xi_l} \pi_{jl} = \left[\pi_l'' + \frac{5\pi_l'}{r} + \frac{\pi_2'}{r^2} - \frac{1}{r^3}\pi_2'\right]\xi_i \xi_j +$$

$$+ \left[\pi_l'' + 4\pi_l + \frac{\pi_2'}{r}\right]\delta_{ij} \quad . \qquad (20)$$

Furthermore, $S_{jk,\, i}$ being symmetrical in indices j and k and solenoidal in i, can be expressed as

$$Sjk,\, i = \frac{2}{r} S'\xi_i \xi_j \xi_k - (rS' + 3S)\left(\xi_j \delta_{ki} + \xi_k \delta_{ji}\right) + 2S\, \xi_i \delta_{jk} \qquad (21)$$

and, therefore,

$$\frac{\partial}{\partial S_k} S_{jk,i} = \left[\frac{6}{r} S' + S''\right] \xi_i \xi_j - [10S + 8rS' + r^2 S'']\delta_{ij} \qquad (22)$$

and

$$\nabla^2 \frac{\partial}{\partial \xi_k} S_{jk,i} = \left[S^{iv} + \frac{12}{r} S'' + \frac{24}{r^2} S' - \frac{24}{r^3} S'\right]\xi_i \xi_j +$$

$$+ \left[-r^2 S^{iv} + 14rS'' - 46S'' - \frac{24}{r} S'\right]\delta_{ij} \qquad (23)$$

and

$$\frac{\partial}{\partial \xi_k} L_{jk,i} = \left[\frac{6}{r} L' + L''\right] \xi_i \xi_j - [10L + 8rL' + r^2 L'']\delta_{ij} \qquad (24)$$

$$R_{ij} = \frac{R'}{r} \xi_i \xi_j - (rR' + 2R)\delta_{ij} \qquad (25)$$

Inserting there in equation (13) we have

$$\overline{A_i' A_j''} = \frac{16}{r^2} \xi_i \xi_j \left(Q'^2 + H'^2\right) + \frac{1}{r}\frac{\partial P(r)}{\partial r}\delta_{ij} +$$

$$+ \left[2\pi_l'' + \frac{5\pi_2''}{2} + \frac{\pi_2''}{2} - \frac{1}{r^3}\pi_2'\right] \xi_i \xi_j +$$

$$+ 2\left[r\pi_l' + 4\pi_i + \frac{\pi_2'}{r}\right]\delta_{ij} - 2\nu\left[S^{iv} + \frac{12}{r} S'' + + \frac{24}{r^2} S'' - \frac{24}{r^3} S'\right]\xi_i \xi_j$$

$$-2\nu\left[-r^2 s^{iv} - 14rs'' - 46s'' - \frac{24}{r} S'\right]\delta_{ij} +$$

$$+\nu^2\left[\frac{8Q''}{r^2}-\frac{8}{r^3}Q'-\frac{16}{r}Q''-10Q^{iv}-rQ''\right]\delta_{ij}+$$

$$+\nu^2\left[\frac{Q^{v}}{r}+\frac{8Q^{iv}}{r^2}-\frac{24Q''}{r^4}+\frac{24'}{r^5}\right]\xi_i\xi_j-$$

$$-\left[\frac{12H'^2}{r^2}+\frac{2H'H''}{r}\right]\xi_i\xi_j+\left[6H'^2+2rH'H''\right]\delta_{ij}+$$

$$+2\nu f\left[\left(\frac{Q''}{r}+\frac{4}{r^2}Q''-\frac{4}{r^3}Q'\right)\xi_i\xi_j-\left(rQ''+6Q''+\frac{4}{r}Q'\right)\right]\delta_{ij})$$

$$-2f\left[\left(\frac{6}{r}S'+S''\right)\xi_i\xi_j-(10S+8rS'+rS'+r^2S'')\delta_{ij}\right]+$$

$$-2f\left[\left(\frac{6}{r}L'+L''\right)\xi_i\xi_j-(10L+8rL'+rL'+r^2L'')\delta_{ij}\right]+$$

$$-2f^2\left[\frac{Q}{r}\xi_i\xi_j-(rQ'+2Q)+\delta_{ij}\right]+f^2\left[\frac{R'}{r}\xi_i\xi_j-(rR'+2R)+\delta_{ij}\right] \quad (26)$$

and consequently, we obtain three independent Scalar equations.

$$\alpha(r,t)=\frac{16}{r}\left(Q'^2+H'^2\right)+2\left[\pi_i'+\frac{5\pi_1'}{r}+\frac{\pi_2'}{r^2}-\frac{\pi_1}{r^3}\pi_2'\right]-$$

$$-2\nu\left[s^{iv}+\frac{12}{r}s''+24\frac{S''}{r^2}-\frac{24}{r^3}S'\right]+$$

$$+\nu^2\left[\frac{Q^{v}}{r}+\frac{8Q^{iv}}{r^2}-\frac{24}{r^4}Q''+\frac{25}{r^5}Q'\right]-\left[\frac{12H'^2}{r^2}+\frac{2H'H''}{r}\right]+$$

$$+2\nu f\left(\frac{Q''}{r}+\frac{4}{r^2}Q''-\frac{4}{r^3}Q'\right)+2f\left(\frac{6}{r}L'+L''-\frac{6}{r}S+S''\right)+\frac{f^2}{f}(Q'-R') \quad (27)$$

$$\beta(r,t)=\frac{1}{r}\frac{\partial P(r)}{\partial r}+2\left(r\pi_i'+4\pi_i+4\pi_i+\frac{\pi_2'}{r}\right)-$$

$$-2\nu(r^2S^{iv}+14rS''+46S''+24S')+$$
$$+\nu^2\left(\frac{8Q^{v}}{r^2}-\frac{8}{r^3}Q'\frac{16Q''}{r}-10Q^{iv}-rQ''\right)+$$

$$+\left(6H'^2+2rH'H''\right)-2\nu f\left(rQ''+6Q''+\frac{4}{r}Q'\right)$$

$$+2f[10(S-L)+8r\,S'-L')+r^2(S''-L'')]+$$
$$+f^2[r(R'-Q']+2(R+Q)] \quad (28)$$

It is possible to write π_1 and $\pi_2$in terms of H as in Chandrasekhar (1955) – namely

$$\pi_1 = -\frac{24}{5}\int_r^{\infty}\frac{H'^2}{Y}dy - \frac{14}{5r^5}\int_o^r H'^2 y^2 dy \tag{29}$$

and

$$\pi_2 = \frac{4}{3}\int_o^{\infty} H'^2 y\, dy + \frac{8}{5}r^2\int_o^{\infty}\frac{H'^2}{y}dy + \frac{14}{15r^5}\int_o^{\infty} H'^2 Y^4\, dy \tag{30}$$

Where the expressions for π_1 and π_2 have been obtained on the assumption that $\overline{u_i' h_j''} = 0$ which means that any tensor having an odd number of components of h is neglected. Thus we obtained expressions for α(r, t) and β(r, t) in terms of Q, H. which are the defining scalars of Q_{ij}, H_{ij} and determine the acceleration covariance from equation (14)

In the case of ordinary turbulence these results are

$$\alpha(r,t) = -\frac{16Q^2}{r} + v^2\left(\frac{Q^v}{r} + \frac{8Q^{iv}}{r^2} - \frac{24}{r^4}Q'' + \frac{24}{r^5}Q'\right) \tag{31}$$

$$\beta(r,t) = -\frac{1}{r}\frac{\partial P(r)}{\partial r} - v^2\left(rQ^v + 100^{iv} + \frac{16}{r}Q'' - \frac{8}{r^2}Q'' + \frac{8Q'}{r^3}\right) +$$

$$4(rQ' + 4Q) \tag{32}$$

Which are the same results as obtained by Kishore and Singh (1984).

Also at sufficiently high Reynolds number viscous dissipation effect is negligible, and we get the simplified expressions for α(r, t) and β(r, t) as

$$\alpha(r,t) = \frac{16}{r}\left(Q'^2 + H'^2\right) + 2\left(\pi_1' + \frac{5\pi_2'}{r^2} - \frac{\pi_2'}{r^3}\right) -$$

$$-\left(\frac{12H'^2}{r^2} + \frac{2H'H''}{r}\right)$$

$$+2f\left[\frac{6}{r}(L' - S') + (L'' - S'')\right] + \frac{f^2}{r}(Q' - R') \tag{33}$$

$$\beta(r,t) = \frac{1}{r}\frac{\partial p(r)}{\partial r} + 2\left(r\pi_1' + 4\pi_1 + \frac{\pi_2'}{r}\right) +$$

$$+ (6H' + 2rH'H'') + 2f\,[10\,(S-L) + 8r\,(S' - L') +$$

$$+ r^2\,(S'' - L'')] + f^2\,[r(R' - Q') + 2\,(R-Q) \;\ldots.\; (2.2. \tag{34}$$

In all those problems in which the pressure plays an important role, it will be desirable to know such a type of statistical correlations at two different points in space. But due to greater complexities involved in hydromagnetic turbulence and lack of experimental information, a comprehensive study of acceleration covariance is difficult at this stage.

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