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Interactive Technologies in Mathematics Classes

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Abstract

Interactive teaching/learning of mathematics implies differentiation in the formation of mathematical concepts, adoption of rules and problem solving. The aim of this paper is to determine, for the mentioned types of learning, common methodological frameworks of differentiated interactive classroom teaching of mathematics. For this purpose, the concept of interactive teaching is first defined, and then, for the development of methodological frameworks and processing models, flexible differentiation is defined and used.

Modern approaches to teaching aim to apply the acquired theoretical knowledge in practice. Interactive contents in teaching are a special type of teaching with the help of computers that are applicable in all disciplines and especially in natural and technical scientific fields in which there is a real need for visualization of the process.

Modern education requires innovative approaches in teaching, which gives interactive teaching special importance in order to achieve more effective pedagogical results. Such interactive teaching is based on the interpersonal cooperative relationship of students in class. It overcomes many weaknesses of traditional teaching. We also analyzed the didactic-methodological features and values of interactive exemplary teaching.

Keywords: Interactive Mathematics Teaching, Flexible Differentiation, Methodological Frameworks, Processing Model.

Introduction

The development of technique and technology imposes the need to improve the technology of education at all levels. The advent of computers and other new teaching aids is causing changes that go towards increasing the quality of the teaching process. The

whole process of modernization of teaching is conditioned by the possession and use of appropriate means, equipment and devices. When procuring funds, equipment and devices, educational institutions engage significant financial resources. Today, knowledge is increasing rapidly, so it is necessary to accept and introduce new forms, methods and means of learning. Classes in which only the classic blackboard and chalk are used are no longer satisfactory. In order for teaching to be more efficient, new multimedia teaching aids are needed, which also satisfy modern didactic-methodological principles of teaching. Maintaining and increasing the quality of education aims to provide opportunities to acquire the necessary knowledge and develop the abilities of students, which they will later apply in further education and work. At the same time, a great responsibility lies on the teachers themselves, who must be able to use modern teaching aids and be up to date with their development. With such means, greater interactivity, engagement and motivation of students is achieved during the teaching process, which contributes to the improvement of the final learning outcomes. Projectors (with computer and projection screen) have found application in many schools in recent years. However, they cannot completely replace the classic school board, because they show the already prepared content, so there is no room for additional activities, explanations and changes. By combining the two previously mentioned approaches to teaching and upgrading them, an electronic interactive whiteboard was created (the name multimedia school blackboard is also used) [1].

With this blackboard, interactivity is added to the ordinary projector and the system becomes a technological replacement for the school blackboard, with plenty of new possibilities. Research in the field of educational practice has shown that the traditional school, based on memorizing and reproducing facts, cannot for the most part develop the competencies needed for the challenges, requirements and pace of modern life. Education must not be immune to today's flood of the presence of technologically advanced devices in everyday life and work, but it must be introduced to its currents in an appropriate measure, in accordance with the curricula, age, characteristics of students and technical possibilities. Technology is a significant resource in expanding and upgrading learning systems. It strives to find new optimal systems of adaptive teaching. The opportunity to offer designed, multimedia, interactive, electronic educational forms, opens a chance for every teacher and student to change the educational process into a new, higher quality dimension. The definition and application of information and communication technology programs raises the issue of curriculum innovation, teacher training and the development of specific teaching methods[2]. Methodical and efficiently used information and communication technology enables us to more clearly and essentially express new contents in the teaching of various subjects. When introducing new information and communication technologies in the teaching process, one should be very careful. Teaching that includes advanced new technologies will have positive effects only if it is confirmed at the same time by both students and teachers. The development of information and

communication technologies has had a great impact on society as a whole, and thus on education. Teachers' needs for the realization of teaching contents have changed [3].

Instead of the frontal type of teaching, a gradual transition to collaborative learning is needed, which supports the active role of students. There is a need for a teaching tool that will enable the display of various multimedia content while providing dynamism, flexibility and interactivity in teaching. One of the ways to introduce interactivity in teaching, to additionally motivate students and to engage all the senses when acquiring new knowledge is to use an interactive whiteboard. The use of a "smart board" is not a guarantee that the class will be successful, and the teaching as a whole is of high quality.

Application of Electronic Interactive Boards

This type of teaching provides better motivation of students and easier maintenance of their attention, thanks to the fact that students in this type of teaching can be more active, instead of concentrating on taking notes. The use of different sources of knowledge greatly enriches the content of teaching and contributes to the development of functional abilities of students [4].

To organize classes in this way, you need: a computer, a projector and a projection surface on which to present, write, draw, etc. You can write on the board with special pencils, in some models with the pressure of a finger, and for the youngest age, writing balls are also designed. Interactive whiteboards work by sending information from the whiteboard to the computer, the computer manages the image on the whiteboard, which is at the same time a canvas on which the image is constantly projected, but also generates feedback to the computer system.

Figure 1. Interactive whiteboard

The use of interactive whiteboards in creating teaching materials allows the teacher to use documents from different programs on the desktop, marking, enlarging text, highlighting with the function of "reflector" and other options, which are used to focus important content, allows him to save all changes made during the lecture or keeping the presentation in its original form so that the next lecture process would go the other way, according to the needs of students.

On interactive whiteboards, with the help of so-called pencils or fingers, you can circle, write or draw different shapes that can be easily recognized as regular geometric shapes as needed, and with simple movements you can arrange the given elements on the surface, ie. boards, which is especially useful in schematics [5].

Also, during the presentation, it is possible to connect with other pages on the desktop, connect with other documents and programs on the computer and connect to the Internet. This possibility provides the necessary dynamics in the teaching process and makes it easy to include numerous examples in the process of explaining the material, which is of great importance for the development of practical knowledge and skills of students.

The introduction of interactive electronic whiteboards in ITS teaching makes lectures and the learning process more attractive, dynamic and meaningful. It enables teachers to be more creative in compiling teaching materials, facilitates the process of storing, modifying and forwarding materials to students, and helps students to become more fully involved in the learning process.

Students no longer have to worry about notes or try to maintain attention, because in this way, above all, their motivation for active participation in the learning process is encouraged.

Characteristics of Interactive Teaching of Mathematics

The learning process is at the core of every human activity. Learning is not just reading textbooks and attending lectures and seminars. Most life situations are more or less a kind of learning. Every time something new is done, knowledge is acquired is the result of lived experience.

In accordance with the modern methodology of teaching mathematics, the degree of interactivity of teaching / learning is the basic criterion for the quality of the process of basic mathematical education. Accordingly, the goal of our research is to create an original and realistically feasible methodology for teaching / learning mathematics, with the greatest possible degree of interactivity. For this purpose, by dominant use of the method of comparative theoretical analysis, we have created the theoretical basis of the research. Using a descriptive-analytical method, we determined the concept and characteristics of interactive teaching, selected and described the most important teaching systems and

methods, forms of work and teaching aids. The methodological approach to interactive teaching, for each class separately, contains original methodological frameworks with models of interactive processing of all teaching units.

The notion of interaction in its broadest sense is defined as an activity that is exchanged between at least two subjects, people or media. Interactive teaching / learning, as one of the most common strategies of modern education, is a narrower concept than the concept of interaction because it primarily refers to the interpersonal relationship. Interaction, on the other hand, implies active relationship or communication of students, teachers, parents and other subjects: "Depending on the nature and type of knowledge, the student in the learning process encounters various intellectual problems and develops specific forms of activities" (Pešikan, A. Ž., Ivić, I. 2000: 164). Through interactive teaching, we make changes in the thinking, emotions and behaviors of students [6]. That is why it is important that the interaction is carried out correctly. The didactic-methodological knowledge so far puts in the foreground the knowledge that is reached with the active participation of students, through personal work and performing experiments. The role of the teacher is to guide, encourage and teach the student how to learn mathematics. In doing so, each student activity needs to be accompanied by teacher feedback (which is a proven psychological need of each individual). In learning mathematics, such work implies first of all the mental activities of the students. The outcomes of such work should increase the effects, such as the optimal development of cognitive and conative abilities, critical thinking, creativity, etc. "Through interactive learning, thanks to social interaction, we make changes in people's thinking, emotions and behaviors" (Milijević, S., 2003, p. 38).

The stated statements refer, first of all, to the mastering of the program contents in the teaching of mathematics, and so far they have not been sufficiently respected. Modern methodology of teaching mathematics requires that the didactic principle of conscious activity of students be given a privileged role in relation to other principles. This would imply the teaching of mathematics in which students are the main and active factors, who participate not only in the teaching process, but also in the choice of work methodology. This increases the motivation of students, their competence and responsibility for their work. In this way, teaching and learning mathematics becomes truly interactive. Interactive learning makes better use of what is learned in new situations of learning mathematics, the transfer of learning is greater, what is learned lasts longer and is remembered. It implies, independently or with the guidance of the teacher, bringing students to knowledge with flexibility in thinking and more expressed creativity. Therefore, for the analysis of each interaction, Suzić in his book (V. Suzić, 2005) proposes four aspects: "cognitive, emotional, target and active" (Ibid.). The methodology of teaching mathematics offers as possible different ways of working, the combination of which can successfully achieve the interactivity of teaching and learning. All modern systems, methods, forms and means are used for the preparation and realization of

interactive teaching. Their choice depends on the age of the students, the goals and tasks of teaching, as well as on the program contents of each topic, ie teaching unit. We have to teach mathematics as an interactional process, in which the student and the teacher are in a cooperative (cooperative) relationship, whereby the student's activity gradually increases[7]. The interactive processing of mathematics teaching units in the younger grades of primary school, approximately, should be performed with the application of the following structure, which we theoretically based, originally constructed and empirically evaluated. The mentioned structure contains the following phases:

1) Interactive repetition of previously acquired relevant student knowledge;
2) Setting up and defining a problem situation or a copy, which provokes the adoption of new knowledge;
3) Interactive processing of planned teaching contents;
4) Completion, unification and generalization of processed contents;
5) Verification summary.

In the second phase of the structure of interactive processing of teaching units, it can be noticed that the application of problem-based and exemplary teaching has a special role in interactive learning. Bearing in mind that both of these teaching systems are sufficiently researched in didactic-methodological theory and practice, in this paper we will present their basic characteristics and point out some aspects of their application in interactive learning and teaching mathematics. Problem-based teaching is especially characterized by demands for greater engagement of students' thinking activities, a little above their abilities. It is extremely important that the segment of teaching (classes) in which problem-based teaching is applied is realized according to the appropriate organizational structure. In this paper, we use a theoretically based, originally constructed, empirically evaluated and recognized in the pedagogical public structure set by Petrović and Pinter: “

1) Creating a problem situation and formulating a problem,
2) Setting hypotheses,
3) Decomposition and problem solving,
4) Analysis of results, drawing conclusions and generalizations,
5) Application of acquired knowledge. ”(See Petrović, Pinter, 2006).

We apply the mentioned structure of problem teaching in interactive processing of more complex mathematical rules and solving mathematical problems. As is well known, the adjective exemplar is derived from the noun exemplar, which means example or pattern. Accordingly, by nominal definition, exemplary teaching can be defined as exemplary or exemplary. Some authors believe that the exemplary teaching should single out the characteristic teaching contents, which are processed in a methodically complete and exemplary way. At the same time, most of the content should be processed by the students themselves, based on a model or with minimal help from the teacher. In that way, the

activity of teachers would be significantly rationalized, and the activity of students would be increased. However, despite the stated advantages, understood in this way, exemplary teaching cannot be applied to a significant extent successfully. In contrast, for interactive teaching of mathematics, especially in the younger grades of primary school, well-chosen and processed specimens are most often used within one teaching unit, and very rarely within a teaching topic. In the case of the application of examples, the analysis of which arouses the interest of students, the interactive processing of the provided teaching contents is freed from excess examples, and the inductive reasoning is enriched with thought activities. Therefore, exemplary teaching especially affects the level and quality of interactivity in learning or forming concepts, as well as in learning mathematical rules.

Innovations in Mathematics Teaching

Modern approaches to teaching aim to apply the acquired theoretical knowledge in practice. Interactive contents in teaching are a special type of teaching with the help of computers that are applicable in all disciplines and especially in natural and technical scientific fields in which there is a real need for visualization of the process [8].

One of the most important goals of teaching mathematics is to teach students to think, that is, to be able to solve problems in future life. Specifically, this refers to the application of acquired theoretical knowledge in practice. In the paper, mathematician George Polya describes a problem-solving model in mathematics that consists of four steps: problem, making a plan, executing a plan, and looking back. We can conclude that research-oriented teaching fits perfectly into the above problem-solving model. The application of interactive content certainly finds application in this model.

Mathematical education software is designed for innovative, interactive and dynamic teaching in various areas of mathematics.

Modern computer technologies have a significant impact on mathematics education, its content, methods and techniques, curricula, as well as on the overall organization of teaching and learning processes. In modern mathematics education, the question is no longer whether computer technologies should be applied, but the main task is to reach optimal solutions on how and in what way to apply new technologies in teaching practice, in order to improve the quality of teaching and make learning more efficient [9].

The application of technology in teaching and learning must be well thought out, with a methodically well-designed plan, an appropriate theoretical background and with respect for the specifics of the subject. Technology should be used widely and responsibly, in order to enrich the process of learning mathematics (NCTM, 2000: 24). The role of teachers in planning, designing and implementing computer-based activities is very important. Teachers have a responsibility to decide when computer technology can

effectively improve learning opportunities and what type of technology is needed to achieve class objectives. Modern technologies enable teachers to more effectively adapt their instructions and teaching methods to the needs of students, to provide creative opportunities that support the learning process, the acquisition of knowledge and skills. The use of computers in the learning process can be of great help to students in calculating, drawing graphs, working with larger data sets, navigating the world of symbols, connecting different representations of mathematical objects, experimenting, making assumptions and checking their correctness. Computer technology allows students to spend more time thinking about mathematical ideas, researching and discovering mathematical knowledge [10]. With the support of computers, students can develop and demonstrate a deeper understanding of mathematical concepts and can engage in more advanced mathematical content than in a traditional teaching environment.

Mathematical representations and visualization in a computer environment

Modern technology and the use of mathematical representations are closely related. Computer technology and mathematical software increase the ability to use and analyze multiple related representations (Yerushalmy & Shternberg, 2001) and are increasingly sophisticated in terms of “multirepresentative abilities” (Heid & Blume, 2008: 58). Technology transforms the possibilities present in mathematical representations (Moreno-Armella, Hegedus & Kaput, 2008). It can help students gain a better understanding of the use of representations and has the potential to build a thorough understanding of mathematical concepts and ideas (Kaput, 1992) [11].

The invention of dynamic interactive technologies changed the infrastructure of mathematical representations. By introducing recognizable dynamic representations and automated linking of multiple representations, mathematical representations have acquired a quantitatively and qualitatively new dimension (Hegedus & Moreno-Armella, 2009; Kaput, 1992; MorenoArmella et al., 2008). The interaction between dynamic representations and mathematical ideas further enhances mathematical communication, and leads to the discovery of pedagogically powerful “synergies between representations and concepts” (diSessa, 2007).

The potential of computer technologies to connect mathematical concepts with visual representations in a way that encourages mathematical reasoning and conceptual understanding is very important for teaching / learning mathematics (Lopez Jr, 2001). For modern educational technology, there are almost no obstacles to visually show any mathematical or real object and to adapt them according to the sensory and cognitive abilities of students. "While real-life objects become abstract when they appear on the screen, mathematical objects that are abstract on the screen become concrete" (Lester, 2000). Visualization of mathematical concepts in multimedia environments provides an opportunity for activities that promote new ideas and ways of thinking (Yerushalmy,

2005), encourages students to be more active, to reason independently, explore concepts and their relationships, solve problems, generate new information and ask new questions (Van Voorst, 1999) [12].

Technology has the potential to visualize the learning process by stimulating more cognitive processes, supporting reasoning, visual articulation of ideas, and dynamic nonlinear (divergent) thought activities (McLoughlin & Krzysztof, 2001). There are a number of different educational software that can be used in teaching math. Most specialized mathematical software, such as dynamic geometry programs and computer algebra systems, allows symbolic, numerical, and graphical operations to be performed, and provides an environment for actively exploring mathematical structures through multiple representations. Computer Algebra Systems (CAS) are programs that enable numerical calculations and manipulation of symbolic mathematical expressions. Most of them also support the visualization of some algebraic objects, with the proviso that the user cannot directly modify graphical representations. Some of the better known are software packages Mathematica, Maple, Derive, MatLab, Scientific WorkPlace. These programs can be used to process the content of algebra, mathematical analysis, analytical geometry, numerical mathematics, etc.

The use of CAS in mathematics teaching allows the learning process to be relieved of routine algorithmic and numerical operations and thus gives more time for a quality interpretation of the problems to be solved, as well as reflection on the results obtained. That is, the use of computers contributes to a comprehensive understanding of the matter, because the practice of procedural skills is carried out more effectively and in a shorter time, and more time can be devoted to the development of conceptual knowledge (Kadijević, 1999). Dynamic Geometry Systems (DGS) are programs that enable the dynamic construction of manipulative geometric objects. and their algebraic representation. However, these programs have poorer opportunities to work with algebraic representations. Programs that can be classified in SBS or have some of the properties of SBS are: GeoGebra, Geometer's Sketchpad, Cabri Geometry, Cinderella, Euclides, GEONExT, etc.

The special significance of dynamic geometric software is reflected in the visualization and dynamic constructions in which objects retain their mathematical properties and relations even after changes. The interactive environment of this type of software creates conditions for quality experimental research work in teaching geometry and processing content about functions [13]. Dynamic visualization provides an opportunity to show students some aspects of mathematics that are extremely difficult, even impossible, to show using pencil and paper. It's hard to imagine how we would display dynamic processes without a computer and how long it would take us to achieve the same goal. In that way, the efficiency of teaching mathematics increases. In addition, opportunities are

created for the introduction of new mathematical content, highlighting the visual aspects of existing program content and modifying the tasks used in their processing. Laborde (2001) suggests that mathematical problems be classified according to the role that dynamic geometric software plays in the teaching process. It highlights four significant roles of this type of educational software:

- Software can facilitate the material aspects of a task without changing it conceptually.
- The software can be used as a “visual amplifier” in order to facilitate observation (eg when identifying the properties of geometric figures or function graphs).
- The software provides special tools that allow students to solve math problems in new ways.
- Dynamic geometric software enables the creation of a new type of mathematical problems that could not be solved without the use of technology (research of dynamic mathematical concepts and processes, such as geometric transformations, functions, limit values, derivatives ...).

Dynamic mathematical software (DMS) is designed to combine in one package certain functions of dynamic geometry software, computer algebraic systems and spreadsheets. Examples of dynamic mathematics software are GeoGebra and GEONExT (Preiner, 2008). Dynamic mathematical software provides many advantages of DGS and CAS tools and at the same time overcomes their disadvantages, so working with mathematical representations is dynamic, interactive and extremely flexible. The possibility of visualization and dynamic connection of multiple representations in a unique way connect geometry, algebra and analysis, and DMS can be used in almost all segments of mathematics teaching and enable the teacher to effectively adapt certain teaching content and teaching methods to individual student needs[14].

Interactive Boards in the Matematile Teaching Process

The electronic interactive whiteboard is a new teaching tool used in teaching. This paper will consider the general concept of this electronic board, without going into the details of individual models of manufacturers (and there are many). Interactive whiteboard equipment includes: computer, projector and surface for design and work (writing, drawing, ..). You can write on the board with special pencils and / or with some models at the touch of a finger. The board itself is connected to the computer via a USB port or wirelessly, via Bluetooth or Infrared [15].

With an electronic multimedia board, it is possible to have at hand a large amount of teaching material: texts, charts, diagrams, films or animations, needed for class work. It is also possible to record all changes to them or the whole lecture. You can freely manipulate any object (line, geometric body, more complex illustration), and using a "sponge" to delete objects is easy. With its characteristics, it, among other things, enables the display

of a large number of educational software, editing documents on the spot using various software applications, etc.

A board that recognizes and can memorize the activities that take place on it, is often called a "smart board", and in fact the information system is involved in the teaching process. The information is sent from the board to the computer, either in the form of a given command, the execution of which is projected and visible on the board in a split second, or for the purpose of further storage of this information. The computer, therefore, controls the image on the board, which at the same time serves as a curtain on which the image is constantly projected and as a kind of feedback generator into the computer system. direct teaching and interactive examination of the whole department. This statement was also given as a recommendation for the implementation of teaching in schools in Great Britain. However, every novelty, even this one, is characterized by good, but also some bad characteristics.

Video projectors have found application with computers and projection screens in many classrooms in recent years. However, despite the undeniable useful value offered by multimedia in combination with a computer and a projector, in many cases we still lack the classic school board, because the content of the lesson and teaching material in the presentations we project is prepared in advance and does not allow to a greater extent additional activities, explanations, changes, ... Over time, there have been ways to insert interactive materials such as java applets, flash animations, which has expanded the range of materials that we can use in presentations and materials that are interactive. If we insert video material as an object of Windows Media Player, we get an object with which we can nicely control the video in our presentations. In addition to certain Plugins, we could and can insert the entire website into our presentation and use it from there. However, despite all the features of Power Point (PP is not the only presentation software used, but it is by far the most common, so we take it as a benchmark), the simplest operations such as drag and drop, as well as manipulation of objects during the presentation process is extremely difficult to realize directly in the presentation, because it relies on macros and behaves capriciously, which would initially discourage most average users. Furthermore, PP already has a built-in ability to take notes in a variety of colors. The software will even offer to save them to your presentation after you try to stop the Slide Show. But we had to do all that sitting at the computer, not from the desktop. The consequence of managing a teacher's actions while sitting at a computer is that such use can be confusing to students, while when a teacher stands for IT, students have no dilemma where the action takes place because they see where the teacher's finger or pen is.

Another problem is that we did not have the possibility of direct manipulation of objects. For example, when we make a note during a presentation, we want to move it a little to the left ... Unfortunately, we can't just do that because we don't have the ability to select, at

least not until we go from presentation mode to save mode, save everything we wrote, select the desired note, move it to the left and restart the presentation from the current slide. How elegant can we call this kind of work? From this need to control the computer and direct manipulation of objects on the desktop (interactivity) while preserving all the possibilities we already had with the computer, projector and multimedia, interactive whiteboards were created. Although the interactive whiteboard is just a new teaching tool used in teaching, it has brought a lot of novelties. If we only take the ability to manipulate the computer directly from the desktop we projected on, we already have an advantage over pure presentation design, because we can now use functionality like Ball Pen or High Point Highlighter directly on the whiteboard. If we leave aside PP for a moment and turn to a large number of software already used in teaching (Geogebra, 3dCabri, MS mathematics, Sketchup, ...), we will be convinced that the possibility of direct manipulation of the commands of these programs is a great advantage over previous ones. image design without the possibility of interaction with software on the desktop. All interactive whiteboards have one set of tools that provide interactivity, ie the sensitivity of the whiteboard desktop to user commands. These tools, in addition to cursor control, allow all or some of the following actions:

- Write notes on the screen, ie on the content currently visible on the computer screen
- Take a photo of part / full screen
- Video recording in steps
- Record audio
- Handwriting recognition
- Shape recognition
- Magnify part of the screen

Some dashboard models also integrate all of these tools into the flipchart software that comes with the dashboard. It is also possible to record all or part of the lecture, in audio or video format. Any object (line, geometric body, more complex illustration) can be freely manipulated, and using a "sponge" to delete objects is simple. Flipchart software arose from the need to eliminate the above problems with existing presentation software in working with whiteboards and make working with interactive whiteboards easier, more beautiful and more elegant. Over time, these software have grown into more serious high-performance applications, coming with large libraries of ready-made teaching materials that are literally at hand: texts, images, charts, diagrams, videos, audio or interactive animations needed for classroom work. Flipchart software allows us, when using an interactive whiteboard, to organize material through slides or pages that are not visible at the same time, but are easy to display again, with or without notes taken during class, with the ability to include multimedia and interactive materials.

Development of teaching with an interactive whiteboard

Teachers who have begun the transition to the use of IT in their work, usually in the early stages of using IT, use it as a tool to implement existing pedagogy. In that context, teachers will use IT as a substitute for the teaching aids used so far (ordinary blackboard, overhead projector, projector), ie they will implement the existing practice using a new method of presentation. Initially, they use the capabilities of IT software that provide various forms of writing to a very limited extent in their work. As self-confidence grows, teachers explore new ways to use IT with new applications, such as PowerPoint. The teacher is then able to use the capabilities of generic software, for example, writing notes and comments on a presentation slide. This type of activity can, of course, also be undertaken by students and supports interactive pedagogy [16].

Steps for teachers to improve their skills in using IT:

- Writing text by hand on the board in a similar way as on a traditional board
- Using pre-prepared text and graphics - making and using flipcharts
- Saving flipcharts for future use
- Using PowerPoint with IT using directly from the whiteboard and use slide note markers
- Use drag and drop techniques with text and graphics on the whiteboard
- Move back and forth between pages to create a more efficient learning sequence
- Import digital photos and audio recordings
- Use hyperlinks to launching various programs
- Preparation and efficient use of the gallery / library of ready resources
- Sharing the gallery and resources with other teachers

As the teacher progresses in the steps and the stated skills, he changes the linear passage through the lesson towards "jumping" where IT technology allows moving through the lesson in different directions. Navigating hyperlinks following different ideas (but without departing from the goal of the lesson) can be modeled and prepared using links to programs, slides, websites, and student and teacher materials available online. As students understand the functioning of hyperlinks and IT technology, the teacher encourages / instructs them to independently create their own versions of "papers" that they will present using IT and that will be part of a lesson and / or classroom activity. These papers are usually students' response to a task set by the teacher on a specific topic in the form of a Power Point presentation with hyperlinks to resources. Students present their works using IT, discussing the results that stand out in the work. The teacher can at any time put a note next to the important parts of the work that is currently being presented in order to direct attention and / or point out the correction of a mistake in the work (or can encourage students to take on that role). Of course, the teacher can plan and get to know the students in advance at the time when the presentation of their work is planned and even coordinate

it in cooperation with the student if necessary before showing and presenting on IT in front of the class. As students discuss the results, teachers can gain a good insight into the knowledge and understanding of individuals or groups of students during these presentations. The presentations are themselves proof of the students' work and can be printed with the teacher's comments if necessary. With the implementation of such activities, the potential of IT becomes obvious and teachers are slowly beginning to change their pedagogical practice in the direction of maximizing the possibilities of IT:

- The teacher introduces the class to the objectives of the lesson and reminds them of the key moments of the lesson.
- The teacher uses the board to present information to students with various resources (audio, video, simulation, animation, pictures, ...) but so that students manipulate these resources.
- The teacher encourages class discussions and follows them with notes on IT.
- Activities such as naming, drawing and creating graphics the teacher undertakes on the spot, on IT
- Arguments and explanations for some points of view are created on IT by students.
- The teacher and / or students write the text on the board, for example the conclusion of a discussion is recorded and suggestions for improvement are analyzed.

The speed and magnitude of the change in the teacher's work depends on the teacher himself. Essentially, just adapting the teacher's work to the realization with the new teaching aid will not bring significant benefits to either the teacher or the students, except for greater motivation during the initial period in which the students are still fascinated by the new technology. Over time, teachers can use their experience and knowledge and, with the introduction of a new teaching tool, develop new strategies and change their own pedagogical practice. Changing pedagogical practice is a process in which teachers need to change the current way of working so as to increase interactivity not only in terms of IT technology, but also the mutual interactivity of teachers and classes initiated by this technology to make full use of this technology in the classroom.

Interactivity and IT

In the physical interaction of users with multimedia materials and resources in working with IT, we single out the six most common operations:

- Drag and drop (Drag and drop)
- Hide and reveal (hide and reveal)
- Coloring, shading and highlighting (Color, shading and highlighting)
- Matching items
- Use of animations, movements and simulations (Movement, Animation, Simulation)

- Giving / receiving feedback (Immediate feedback) As key reasons for the positive impact of interactivity on successful learning and teaching using IT technology we can single out:
- Obviousness. Well-conceived material and user interaction with objects on the board using a pen, finger or some other device have a positive effect on understanding.
- Game elements. Elements of the game make the use of this board fun for students, and when something is fun it is easier to learn and better remembered.

Integrating sound, animation, video and text and other interesting resources into the lesson has a positive effect on attention and learning. Elements of the game can appear in various forms when it comes to IT, as a computer game with educational potential, as a designed game of knowledge competition between students in groups divided into groups, ...

The interaction exists between players and the board, the whole class and teachers as game leaders, team members, ie students, ...

- **Visualization**

The use of colors, movements, accents and other visual effects has a positive effect on learning in all students. Students' interaction with the visual world of the media is intuitive and effective.

- **Student work**

The wide range of possible media materials and the possibility of manipulating them is a big plus for IT technology.

The creation of multimedia materials on a given topic by students stimulates the mutual interaction of students through discussion and direct manipulation of the media. This interaction between students has a positive effect on learning. In this case, the teacher is the mentor during the drafting and the mediator of the discussion.

Interactivity in teaching with IT is not a one-way process but a process where the teacher modifies his approach to the needs of the students. For successful learning, there must be student interaction with the teacher, resources, and other students. By interaction we mean not only the physical use of the board by students, but also interaction in terms of exchanging opinions and ideas with other students, teachers, ... The use of IT certainly encourages more active participation of students in the realization of classes. What is certainly important is that the teacher must carefully select and create resources and manage the development of the class to avoid negative connotations of active student participation. The teacher should create resources so that they present a challenge to the students but so that the tasks set before the students are solvable [17].

Mathematical Software Details

Mathematical software is software used to model, analyze or calculate numeric, symbolic or geometric data. Many mathematical suites are computer algebra systems that use symbolic mathematics. They are designed to solve classical algebra equations and problems in human readable notation [18].

By mathematical software we understand computer programs that implement mathematical objects and relations and which are used to analyse, solve, simulate or model a mathematical problem. It usually relies on mathematical publications that provide methods and algorithms describing a concrete path to solve a mathematical problem. Software represents, often quite deep, mathematical knowledge in an active way, including automated mathematical reasoning and conclusions [19]. Therefore, mathematical software has its own characteristics that differs substantially from mathematical literature:

- Mathematical software has often experimental character.
- It is focused more on modelling, simulating and solving mathematical problems rather than on structural description of mathematical concepts and theories.
- Mathematical software is of dynamic nature and a 'living' object under permanent development by improvement and extension. It can 'die' and lose its usefulness, in contrast to mathematical publications which remain valid for ever.
- Implementation of mathematical software depends on the environmental features: hardware, operating system, programming language, interfaces, other software.
- Hence, mathematical software cannot be combined in a free way as this is valid for theoretical results. Therefore, the software must provide appropriate interfaces.
- Quality of mathematical software can't be simple evaluated as 'correct' or 'non-correct', it also depends on a lot of other factors such as performance, ease of use, operating system and programming language, and special features. The evaluation of the quality of software is a difficult and complex problem.
- Granularity of software varies from big general purpose systems or libraries to specialised and small packages.

Mathematical software packages can approximate solutions to a large range of problems in mathematics, including matrix equations, nonlinear equations, ordinary and partial differential equations, integration, and optimization.

Geometry Pad

This mathematical tool will help you in learning geometry and allows you to practice vital constructions. This tool will act as your personal assistant in learning geometry. This is a student-friendly tool that helps in the presentation of geometric constructions, taking

measurements, compass use and experimentation with different geometric shapes in an easier manner [20].

The tool is beneficial for both the teacher as well as students. Teachers can take the help of this tool to provide a good understanding of various geometric concepts. This tool can provide assignment help to students as well, they can learn geometry with the help of this tool by sitting at home. So, if you are looking to get hassle-free solutions to your geometry problems, grab this tool online and reap all the benefits.

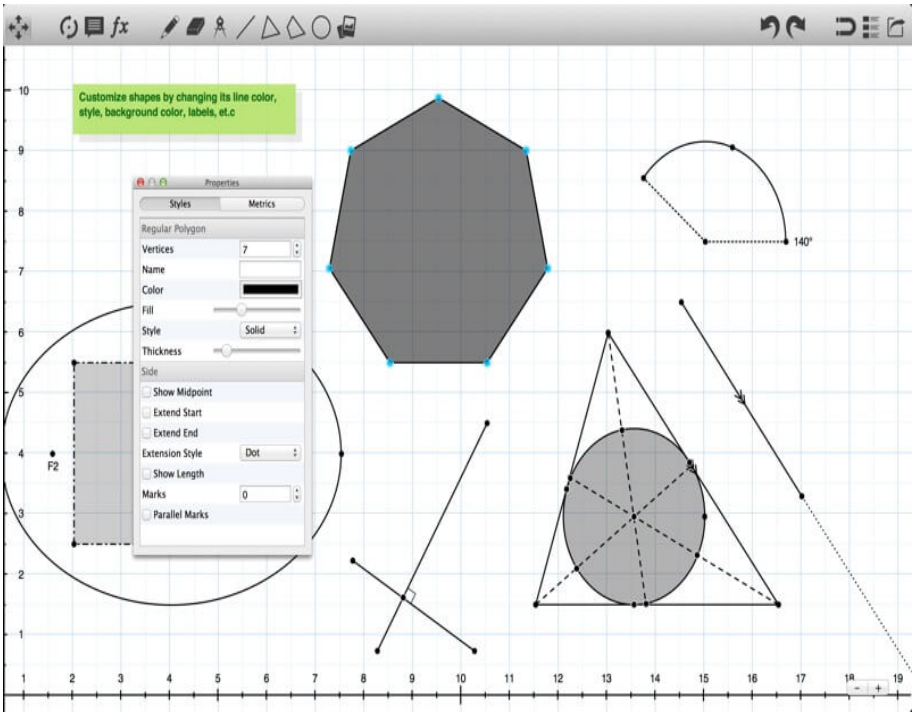

Figure 2. Mathematical tool Geometry Pad

Math Editor

If you are having issues in solving the math equations, use math editor. This is a perfect solution for college students who are facing issues in math equations. This software helps you to form equations on screen by using Greek symbols, alpha, beta, square root, and other symbols in a quick and easy manner.

One can also edit and save the equation in real-time. This mathematics software is one of the most student-friendly advanced math software. It allows you to save equations in the form of an image file that can be used in the MS office documents, web, and paint. Therefore, if you are looking for a personal helping hand all the time then this software can be a blessing for you [21].

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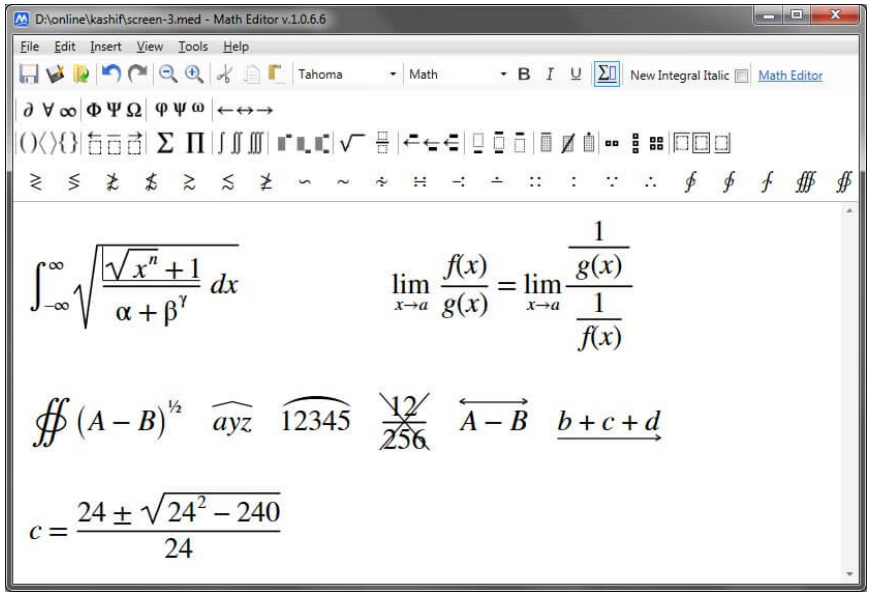

Figure 3. Software for solving mathematical equations

SpeQ Mathematics

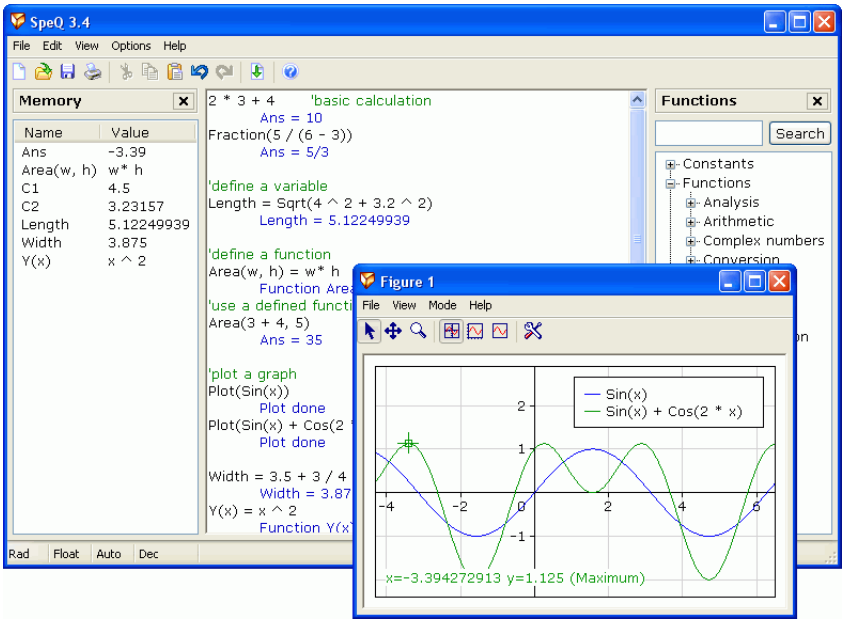

Figure 4.The program SpeQ Mathematics

Learn math and solve complicated mathematical problems easily with SpeQ Mathematics. It is a freeware that comes with embedded support for a wide range of constants, mathematical functions, and variables. While it helps you define custom variables and functions, it also allows you to solve trigonometry problems. It comes accompanied with a simple interface that features sheet of calculations and where you can add, edit and carry out calculations.

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The enVision Platform

Overview

The enVision interface is what a typical user will see and interact with. It loads into a standard Web browser with no installation, and helps users input and communicate their ideas. It has two communication components: a shared workspace, where any number of participants interact with each other in a mathematical discussion, and a text chat window, where most of the other discussion occurs. The shared work space is essentially a whiteboard drawing program that has special drag-and-drop resizeable math

symbols, as well as a panel of special math characters that can be placed with ease onto the whiteboard. Everything that is written on the whiteboard and the chat area is visible to all participants. MathServer is the application that relays the information between different users to ensure that their enVision interfaces are synchronized, thus creating and maintaining a shared workspace [22].

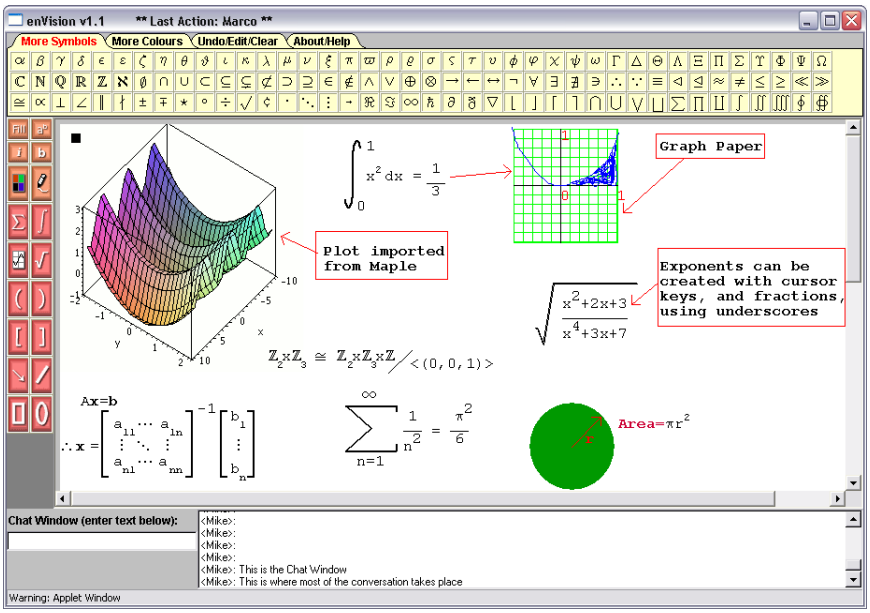

Figure 5.Student Mode view of enVision interface

The core philosophy of enVision is simplicity and ease of use, i.e., allowing for the communication of mathematical content as easily and quickly as possible with a minimal learning curve, for both student and instructor, and minimal hardware and software requirements. The emphasis is on communication in a quick and intuitive fashion, rather than having typeset letter-perfect documents.

In order to keep the program platform independent and accessible to low-bandwidth users, enVision supports a core set of functionalities. It makes it possible to load JPEG and GIF images, which could easily be extracted from applications like Maple, or from PDF

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documents, with freely available programs like ImageMagick. Likewise, it is assumed that if video or audio are desired features, appropriate packages could be loaded simultaneously. Although enVision is easy to install and maintain, it also has advanced features that allow for integration and customization into existing environments such as Blackboard and WebCT. In short, the program can be easily installed and running in minutes as is, but it is also flexible and can be customized by those who are inclined to do so.

enVision Web Interface

Most of the interface options are accessed through the orange/pink button panel on the left-hand side of the interface or the tabbed panels on top of the applet.

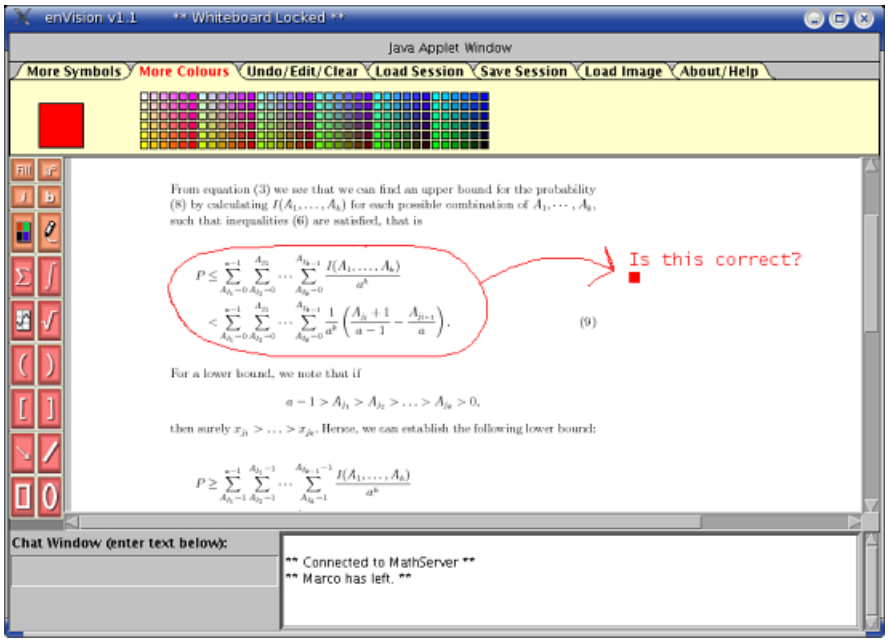

Figure 6. Instructor Mode view of enVision interface displaying PDF

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enVision Web Interface

Table 1. Functions of the main buttons on the enVision interface

Button(s)	Function	Button(s)	Function
Fill	Toggle fill mode on/off for ovals and boxes	a°	Toggle text size (useful for superscripts and subscripts)
i b	Toggle italics and bold text on/off		Change color (cycling through black, red, green, and blue)
T	Toggle between sketch and text mode	Σ ∫ √	Resizeable drag and drop sum, integral and root symbols
	Resizeable drag and drop graph paper	( )	Resizeable drag and drop parentheses
[ ]	Resizeable drag and drop brackets for matrices	↘	Resizeable drag and drop arrow for pointing
	Line drawing tool		Rectangle and oval drawing tools for drawing both filled and unfilled shapes

- The small orange buttons are essentially toggle switches, which toggle fill mode (for rectangles and ovals), subscript text (on/off), italicized text (on/off), and bold text (on/off).
- The first large orange button cycles through four colors (black, red, green, and blue), while the second large orange button toggles text and sketch modes, which is particularly useful if you are on a tablet PC.
- The pink buttons are for drawing the following resizeable objects: sums, integrals, graph paper, roots, parentheses (left/right), brackets (left/right), arrows, lines, rectangles and ovals.
- The tabbed panels assist in editing and provide easy access to additional symbols and colors. Keyboard shortcuts are also described in the edit panel.

One of the key features of enVision is the behavior of the cursor in text mode. The cursor keys move the cursor in half-steps to allow for the easy creation of exponents, subscripts, etc. By clicking the mouse or using the <F1> key, users can set alignment marks for the Enter key to line up text. The Backspace key also acts as an undo key for both text and symbols.

Finally, additional information regarding which user is using the whiteboard component is displayed on the title bar of the enVision window (when the whiteboard is locked, the title bar displays its locked status).

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Interactive Learning Brings Better Grades in Mathematics

Interactive learning and teaching is a modern method of work suitable for all ages. This efficient way of presenting mathematics implies the use of advanced technology in teaching and provides better results and better grades compared to traditional teaching.

During the ‘60s, lecturers and experts found that schoolchildren learn best through a combination of passive and interactive learning. Unlike the so-called passive learning represented in traditional teaching, during which students only observe the learning process and listen to information, interactive learning requires an answer from them, ie. active participation in teaching.

Learning opportunities are independent of space and time, that is, students can learn when and where it suits them.

OK mathematics is an innovative pedagogical and technological solution that helps your children learn, understand and master mathematics in a simple and interesting way. The technology platform, developed according to the most modern standards, enables teaching on the Internet, interactive contact with professors, time planning and scheduling classes.

Instead of passively listening to lectures in classrooms, students very quickly acquire the necessary knowledge in appropriate ways.

It is known that the traditional approach is dominated by the teaching approach, while interactive teaching allows teachers to bring the material of the chosen class closer to your child in an accessible, modern and gradual way. Video lessons, doodle video form will present a certain mathematical theorem in a fun way and explain to the student how to solve a certain problem.

Multimedia enriched learning is achieved through didactically shaped learning materials.

Tasks adapted to the material and program of mathematics, explained through interesting video works, will enable your child to understand how mathematical rules and theorems are applied. In each area, in addition to lessons, assignments and videos, as an additional source of information, interesting Khan Academy video lessons await you, which are intended for students who want to learn more and hear some interesting things from the area they are going through. When you work through a certain area, an online test awaits you to test your entire knowledge in that area.

The efficiency of interactive learning is also achieved through continuous feedback through advanced technology.

Through each part of math classes, online classes, progress tests and consultations, you will have the help and selfless support of professors with many years of experience in

primary, secondary and higher education. They will guide you, answer your and your child's questions, solve their dilemmas and help them master the material more easily, all in a very short time (usually within 24 hours). In addition, your child can chat online with the lecturers as if they were in a classroom.

Interactive learning will help your child prepare more adequately for, sometimes, a harsh life. Engaged students who actively participate in their own education often have a habit of using newly acquired skills and knowledge in various life situations. The effects of interactive learning are manifested through a greater degree of development of cognitive, emotional, social and work-action competencies of students.

Mathematics is not an awkward system of formulas, expressions and unsolvable problems, but a very logical system that you can easily master when it is explained to you in the right way. Interactive learning is the most important step towards making your child understand and love math.

Conclusion

Modern mathematical education, its harmonization with the requirements of science and technological development and progress, permanently requires the introduction of innovations that contribute modernization, rationalization and efficiency of the teaching process. In order to be comprehensive perceiving and solving problems related to teaching and learning mathematics, researchers mathematical education pay significant attention to mathematical representations, visualization and modern educational technologies, their role and significance for the process learning. Multiple representations, visualization and educational technology, recognized as necessary components of mathematics education, it is necessary to implement them in everything segments of teaching, because of their potential to promote mathematical insight and understanding and improve the learning process. These aspects are especially important for the study of functions, fundamental concept of mathematics teaching.

One of the prominent goals of teaching mathematics is the development of general and interdisciplinary competencies that are needed to understand the phenomena and laws in nature and society and which will to enable pupils / students to apply the acquired knowledge in solving various ones tasks from life practice and for successful continuation of mathematics education (PNPPG, 2011). Apart from mathematics, functions and their graphical representations are also used in others scientific disciplines, as a means to describe, explain, explain scientific concepts, phenomena and processes, confirm or anticipate. Application of functions, especially their graphical representations as mathematical model for solving problems from other scientific disciplines and various problems in a real environment, provides an opportunity to correlate with the teaching of mathematics other subjects and that pupils / students through the application of knowledge

from one scientific disciplines in the second acquire comprehensive, lasting and functional knowledge.

The key aspects of the original innovative approach are: visualization, multiple representations, conceptual knowledge and a gradual transition to higher levels of abstraction in the development of mathematical thinking.

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