International Journal of Transformation in Applied Mathematics & Statistics
Vol. 3, Issue 2 – 2020
ISSN: 2581-7620

# Comparison of Univariate and Bivariate Time Series Forecasts of Nigerian Stock Exchange variables

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## Abstract

The study examined univariate and bivariate time series forecast of Nigerian stock exchange variables: All Share Index (ASI) and the External Reserves (ER) which comprise of monthly value from 1985 to 2018 of them both. It filled the lacuna by explicitly modeling and forecasting stock returns in Nigeria using the univariate ARIMA and bivariate VAR models. The monthly and yearly means plots were done, to have a better understanding of the series behaviours. The order of the regular autoregressive and moving avenge model that is necessary to adequately represent the time series model was determined. The series plots showed that ASI series is integrated of order 1 without seasonality while ER series is integrated of order 1 with the seasonality of order 12**.** A suitable ARIMA and VAR Model were obtained for both series using model selection criteria (MSC) and the models were used to generate forecasts. The univariate and bivariate model forecasts were compared and the result shows that the bivariate model is better to predict the two series than the univariate model from the result of forecast accuracy measures (i.e. MAPE and MSE).

**Keywords:** Nigerian stock exchange variables, ARIMA Model, Vector Autoregressive (VAR) Model, Forecast Accuracy Measure, Model Selection Criteria.

## 1. Introduction

Forecasting is a very global important part of econometric analysis. How do we forecast economic variables, such as gross domestic product (GDP), inflation, exchange rates, stock process, and unemployment rates?. Other economic variables problems involved in forecasting prices of financial assets, such as stock process and exchange rate are of great concern (Ali, 2013). It is no longer news that the global economic crisis and COVID 19 pandemic has brought about a shortage of financial resources and a general downtown in stock prices across the globe. Therefore forecasting stock prices will help provide a way to expect and maybe avoid the risk of a large change in prices.

Time series analysis is a statistical technique that deals with time series data, or trend analysis. Time series data means that data is in a series of particular periods or intervals. The data is considered in three types: (1) Time series data: A set of observations on the values that a variable takes at different times. (2) Cross-sectional data: Data of one or more variables, collected at the same point in time. (3) Pooled data: A combination of time series data and cross-sectional data. The Stock exchange market (All Share Index (ASI) and External Reserve (ER) series) has become one of the well-known investments in the recent past due to its higher returns. It has become a great part of the global economy as the exchange market influences both the personal and corporate lives and the economic life of a country. The Nigerian stock market forecasting is known more by its failure than success since its prices reveal the judgment and what investors expect base on the available information. Base on this, the accuracy in forecasting the stock market prices or predicting the trend accurately is of importance for anyone who wishes to invest in the dynamic global economy. This study exclusively deals with the time series forecasting model and in particular the Autoregressive Integrated Moving Average (ARIMA) models which were described by Box-Jenkins. The paper considered the components of the Nigerian stock exchange (ASI and ER) which comprises monthly and yearly values for the long period of January 1985 to December 2018.

The aim of the paper to compare the forecasts obtained from time series models (ARIMA and VAR model) using accuracy measures of forecast values for optimum economic decisions. The specific objectives of the study are to (1) describe the series plots, yearly mean plots, and monthly mean plots and obtain the stationarity of the series. (2) determine the year with the highest ASI and ER rate. (3) obtained a suitable model to fit the Nigerian stock exchange series (All Share Index and External Reserves). (4) obtained the forecasts from the obtained models considered (ARIMA and VAR model) covering 2018-2020 using accuracy measures of forecast values.

The paper is limited to ARIMA and VAR models, thus ARIMA model pointing to a single time series given that the major objective of time series economic modeling is to identify the relationship between econometrics variables and use them to estimate and forecast the variables.

## 2 Literature

Over the years, economists and financial analysts have constantly maintained that a market price that is not regulated is the best and stick to prove the true scarcity of a commodity or its worth. It is easy for one to evaluate the Nigerian stock market (NSM) performance by the use of a stock market index or returns. The stock market returns can be predicted from a variety of financial and macroeconomic variables which has been an attraction for equity investors.

 

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The stock market index has attracted great attention as a way of measuring a sector of the stock market. The investing public has to a large extent an important indicator used by a benchmark by which investor or fund management compares the returns of his portfolio (Senol, 2012). A stock market index is a tool used by investors and financial managers to describe the market and to compare the return on a specific investment. A stock index is a method by which the value of a section of the stock market is measured. A market index tracks down the performance of a particular basket of stocks considered to stand in place of a particular market sector of the Nigerian economy. Thus, the need to predict the stock price to meet the basic objectives of operators and investors of the stock market for gaining more benefits cannot be overemphasized. This issue has brought to focus the attention of statisticians and researchers all over the world. The stock market is affected by numerous factors and this has created high controversy in the field.

ARIMA modeling has been successfully used in various stock-market activities (e.g price indices, migration rate, rate of currency exchange, etc). Naylor *et al.*, (2012) examined the ARIMA model in contrast to the Whartons econometric model and it was revealed that ARIMA models were better and accurate in forecasting than that of Whartons models. A comparison of values and prices of commodities has always been a difficult task in economic situations. Many authors among whom are: Lirby (2007), Malkeil (2013), and Durbin (2012) have compared, estimated, and forecasted for the future stocks and commodities dealing with auto-correlation.

A successful time series forecasting depends on an appropriate model fitting. A lot of efforts have been done by researchers over many years for the development of efficient models to improve forecasting accuracy. As a result, various important time series forecasting models have been evolved in literature. One of the most popular and frequently used stochastic time series models is the Autoregressive Integrated Moving Average (ARIMA). ARIMA model has subclasses of other models, such as Autoregressive (AR), Moving Average (MA), and Autoregressive Moving Average (ARMA) models. For seasonal time series forecasting, Box and Jenkins (1976) had proposed a quite successful variation of ARIMA model, viz. the Seasonal ARIMA (SARIMA). The popularity of the ARIMA model is mainly due to its flexibility to represent several varieties of time series with simplicity as well as the associated Box-Jenkins methodology for the optimal model building process.

Forecasting stock returns is an important approach to understanding future stock price behavior of the Nigeria Stock Exchange of the All Shares Index. Accurate stock price forecasts would not only reduce uncertainty in stock prices but also provide a way to form expectations and perhaps avoid the risk of a large adverse change in stock prices. The stock price forecast is important for deciding both the timing of stock investment and the relative investment desirability among the various sectors in the market (Fischer and Jordan, 2005). According to Sims (1980), if there is true simultaneity among a set of variables, they should all be treated on an equal footing; there should not be any a priori distinction between

 

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endogenous and exogenous variables. It is the spirit that Sims developed his VAR model. The Vector Autoregressive (VAR) model is an approach in modeling dynamics among a set of variables. The approach usually focuses on the dynamic of multiple time series. Vector Autoregressive(VAR) model is also an independent reduced form dynamic model that involves constructing an equation that makes each endogenous variable a function of their past values and past values of all other endogenous variables.

## 3. Materials and Methods

The study seeks to compare univariate and bivariate time series forecast of the Nigerian stock exchange, using the univariate and bivariate ARIMA and VAR model. The Box-Jenkins approach of model identification, parameter estimation, and diagnostic checking will be adopted in the analyses. The paper is restricted to the Nigerian Stock Exchange. It is also restricted to ASI and ER data. The secondary data used for the study were collected from the Central Bank of Nigeria (CBN) Statistical Bulletin. It is the monthly data of All Share Index (ASI) and External Reserve (ER) on the Nigeria Stock Exchange ranging from 1985 to 2018.

### 3.1 Stationary and non-stationary time series

A time series is said to be stationary if the statistical property e.g. the mean and variance are constant through time. If for $n$ values of observations $x_{1,}x_{2,}x_{3,\ldots}x_n$ of a time series that fluctuate with constant variation around a constant mean μ, then the time series is stationary and all processes that do not possess these properties is called "non-stationary". A non-stationary time series can be made stationary by transforming the time series into a series of stationary time series values (differencing).

### 3.2 Mixed Autoregressive moving average (ARMA) model

Box and Jenkins (1976), noted that the mixed autoregressive moving average model is the combination of $MA(q)$and $AR(p)$. Let's say that $X_t$ is the deviation from the mean $\mu$, then $ARMA(p,q)$ model can be written as

$$x_t - \phi x_{t-1} - \phi_2 x_{t-2} - \cdots - \phi_p x_{t-p} = \varepsilon_t - \theta\varepsilon_{t-1} - \theta_2\varepsilon_{t-2} - \cdots - \theta_q\varepsilon_{t-q}. \quad (3.1)$$

Thus,

$$\phi(B)x_t = \theta(B)\varepsilon_t \quad (3.2)$$

The equation (3.2) can be written as

$$x_t = \phi^{-1}(B)\theta(B)\varepsilon_t$$

$$= \frac{\theta(B)}{\phi(B)}\varepsilon_t = \frac{1-\theta_1 B - \cdots - \theta_q B^q}{1-\phi_1 B - \cdots \phi_p B^P}\varepsilon_t \quad (3.3)$$

 

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The ARIMA model is based on prior values in the autoregressive terms and the error made by the previous prediction. The order of ARIMA model is given by $p, d, q$ where, $p$ represents the autoregressive component, $d$ stands for the differencing to achieve stationarity and $q$ is the order of the moving average.

Seasonal Autoregressive Integrated Moving Average (SARIMA) model applies to time series with seasonal and non-seasonal behavior. SARIMA model has a multiplicative and additive part. The multiplicative is so applied because of the assumption that there exists a significant parameter resulting from the multiplication between nonseasonal parameters. By the use of $\nabla$ and $B$ notation, ARIMA (p, d, q) model can be written as

$$\phi(B)w_t = \theta(B)\varepsilon_t \tag{3.4}$$

*where* the polynomial in $B$ is given as

$$\phi(B) = 1 - \phi_1(B) - \cdots - \phi_p B^p \text{ and } \theta(B) = 1 - \theta_1(B) - \cdots - \theta_q B^p$$

The paper focused on the multiplication model because of the assumption that there is a major parameter between the non-seasonal and seasonal models. This is denoted by $ARIMA\ (p, d, q) \times (P, D, Q)$ written as

$$\phi_p(B)\phi_P(B^s)\nabla^{\mathrm{d}}\nabla_s^{\mathrm{D}}\mathrm{z_t} = \theta_\mathrm{q}(\mathrm{B})\theta_\mathrm{Q}(\mathrm{B^s})\varepsilon_\mathrm{t} \ (3.5)$$

*where*

$$\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \cdots \phi_p B^p; \Phi(B) = 1 - \Phi_1, sB^s - \Phi_2, sB^{2s} - \cdots - \Phi_p, sB^p$$

$$\nabla^d = 1 - B - B^2 - \cdots - B^d; \nabla_s^D = 1 - B^s - B^{2s} - \cdots - B^{2D}$$

$$\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_q B^q \text{ and } \Theta(B^s) = 1 - \Theta_1, sB^s - \Theta_2, sB^{2s} - \cdots - \Theta_Q, sB^{QS}$$

where $z_t$ is the time series at period $t$, $\varepsilon_t$stands for the white noise, $B$ represents the backshift operator, $S$ is the duration of the seasonal model which could be weekly, quarterly, or yearly, $p$ is the autoregressive parameter, $P$ is the seasonal autoregressive parameter, $d$ is the order of the monthly difference(quarterly difference), $D$ is the order of seasonal difference, $q$ is the moving average parameter and $Q$ is the seasonal moving average parameter.

Box and Jenkins (1976) proposed four steps in developing a linear time series which are Model identification, Estimation of parameters, Diagnostic Checking, and Forecasting

## 3.3 Vector Autoregressive (VAR) Model

The basic $p$-lag Vector autoregressive VAR$(p)$ the model has the form.

$$Y_t = C + \Pi_1 Y_{t-1} + \Pi_2 Y_{t-2} + \ldots + \Pi_p Y_{t-p} + \varepsilon_t \quad t = 1, \ldots, T. \tag{3.6}$$

 

*where*

$Y_t = (y_{1t}, y_{2t,...,} y_{nt})$ is an $(n \times n)$ vector of time series variable,

$\Pi = (n \times n)$ coefficient matrices

$\varepsilon_t$ is an $(n \times 1)$ unobserved zero mean with white noise vector process (serially uncorrelated and independent) with invariant covariance matrix Σ

The model can be written in the matrix form as

$$\begin{pmatrix} y_{1t} \\ y_{2t} \\ . \\ . \\ y_{nt} \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \\ . \\ . \\ c_n \end{pmatrix} + \begin{pmatrix} \pi_{11}^1 \pi_{12}^1 \cdots \pi_{1n}^1 \\ \pi_{21}^1 \pi_{22}^1 \cdots \pi_{2n}^1 \\ . \quad . \quad \cdots \quad . \\ . \quad . \quad \cdots \quad . \\ \pi_{n1}^1 \pi_{n2}^1 \cdots \pi_{nn}^1 \end{pmatrix} \begin{pmatrix} y_{1t-1} \\ y_{2t-1} \\ . \\ . \\ y_{nt-1} \end{pmatrix} + \begin{pmatrix} \pi_{11}^2 \pi_{12}^2 \cdots \pi_{1n}^2 \\ \pi_{21}^2 \pi_{22}^2 \cdots \pi_{2n}^2 \\ . \quad . \quad \cdots \quad . \\ . \quad . \quad \cdots \quad . \\ \pi_{n1}^2 \pi_{n2}^2 \cdots \pi_{nn}^2 \end{pmatrix} \begin{pmatrix} y_{1t-2} \\ y_{2t-2} \\ . \\ . \\ y_{nt-2} \end{pmatrix} + \cdots +$$

$$\begin{pmatrix} \pi_{11}^p \pi_{12}^p \cdots \pi_{1n}^p \\ \pi_{21}^p \pi_{22}^p \cdots \pi_{2n}^p \\ . \quad . \quad \cdots \quad . \\ . \quad . \quad \cdots \quad . \\ \pi_{n1}^p \pi_{n2}^p \cdots \pi_{nn}^p \end{pmatrix} \begin{pmatrix} y_{1t-1} \\ y_{2t-1} \\ . \\ . \\ y_{nt-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ . \\ . \\ \varepsilon_{nt} \end{pmatrix} \quad (3.7)$$

## 3.4 Model Selection Criteria (MSC)

The AR and MA order p and q have to be determined by examining the regular and seasonal autocorrelation and partial autocorrelation function; ACF, PACF, SACF, and SPACF for $Y_t$. before an ARMA(p, q) is estimated. The idea is to fit all ARMA(p,q) models with order $p \leq p_{max}$ and $q \leq q_{max}$ and choose the value of p and q which minimizes some model selection criteria. For ARMA(p, q), the model selection criteria are given by

$$MSC(p,q) = Ln(\sigma^2(p,q)) + c_T.\varphi(p,q) \quad (3.8)$$

*where* $\sigma^2(p,q)$ is the MLE of var($\varepsilon_t$) $c_T$ is a sequence indexed by the sample size T, and $\varphi(p,q)$ is a penalty function that penalizes large ARMA(p, q) model.

## 3.5 Information Criteria

The three most common information criteria for selection models are the AkaikeInformation Criteria (AIC), Schwarz-BayesianInformation Criteria (BIC), and Hannan-QuinnInformation Criteria.

## 3.6 Akaike Information Criteria

The AIC is a measure of the relative goodness of fit of a statistical model. The AIC value is given by

 

$$AIC = \mathrm{T}\ln\left[RSS/\mathrm{T}\right] + 2p \tag{3.9}$$

*where* T is the number of data points (observations); ln is the natural logarithm; RSS is the residual sum of square ($\sigma^2$) or the error variance of the model which is an unbiased estimator of the true variance and p is the number of parameters in the model. (Akaike, (1983))

## 3.7 Schwartz-Bayesian Information Criteria (SBIC or BIC)

The BIC is a model selection criteria that involves selections among a finite set of models. The BIC is given by

$$BIC = \mathrm{T}\ln\left[RSS/\mathrm{T}\right] + p\ln(\mathrm{T}) \tag{3.10}$$

*where* the parameters are defined as previous Equation (2.10)

## 3.8 Forecast Accuracy Measures (FAM) of the Estimated Values

To gauge the accuracy of our estimates, the estimated errors will be used to compare the two models forecasts. This is done by subtracting the estimated forecast values (EFV) from the original values or [actual values (AV)] to obtain the estimate errors. The estimated error is denoted by

$$e_i = \mathrm{AV}_i - EFV_i\,,\ i = 1, 2, \ldots v \tag{3.11}$$

*where* v is the number of forecast values

Then accuracy measures considered in this paper are: Mean Error (ME), Mean Absolute Error (MAE), and Mean Square Error (MSE).

### 3.8.1 Mean Error (ME)

The first descriptive Statistics of Error used is called the Mean Error. It indicates the deviation between the actual values and estimates, Mean Error is given as

$$\mathrm{ME} = \left[\frac{1}{v}\sum_{i=1}^{v} e_i\right] \tag{3.12}$$

### 3.8.2 Mean Square Error (MSE)

MSE also indicates the fluctuations of the deviations and it can be calculated as

$$\mathrm{MSE} = \left[\frac{1}{v}\sum_{i=1}^{v} e_i^2\right] \tag{3.13}$$

 

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### 3.8.3 Mean Absolute Percentage Error (MAPE)

This accounts for the percentage of deviation between the actual values and estimates. This can be obtained as

$$\text{MAPE} = 100 \times \left[ \frac{1}{v} \sum_{i=1}^{v} \left| \frac{e_i}{AV_i} \right| \right] \quad (AV_i \neq 0) \tag{3.14}$$

## 4. Results and Discussion

The section is divided into four parts: (1) Plot description and stationarity; (2) ARIMA model identification, (3) the VAR model identification, and (4) ARIMA and VAR model forecast comparison.

### 4.1 Monthly and Yearly Means Plots, the Series of the Data Sets

The monthly and yearly means of the series plots (All Share Index and External Reserves) were done to examine the relationships, trend component, and seasonality effect, if present in the data sets. Figure 4.1 shows the monthly means behaviour of the ASI, where the peak is in June, and the least exchange rate is in January. Also, the monthly mean series shows an increase from the beginning (or swing upward); January to June. Then, randomly show a downward movement from July to December. The yearly means plot in Figure 4.2 shows an upward trend and then downward movements in a random manner. There seem to be evidence of a peak in the year 2007 and depressions almost all through the early periods. ASI series in Figure 4.3 shows an upward trend from the year 1985 to 2007, then randomly shows downward movements from 2007 to 2018. Figure 4.4 shows the monthly means behaviour of the ER, where the peak is in March, and the least external reserves rate is in December. The yearly means plot in Figure 4.5 is similar to Figure 4.2, which shows an upward trend and then downward movements in a random manner in the last years. The ER series in Figure 3.6 is similar to Figure 4.3, which shows an upward trend from the year 1985 to 2007, then randomly shows downward movements from 2007 to 2018.

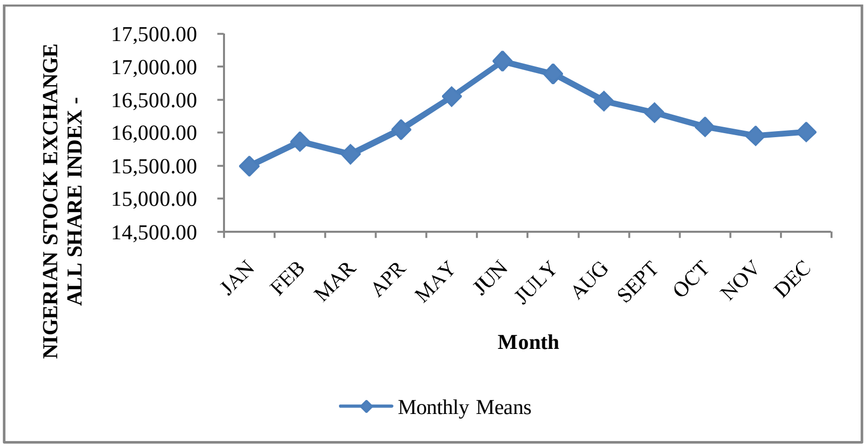


**Figure 4.1.Monthly Means Plot of All Share Index**

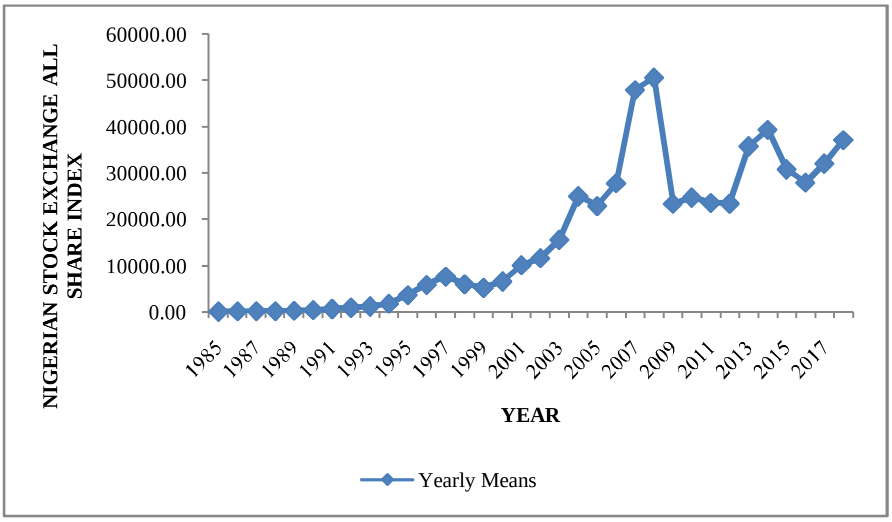


**Figure 4.2.Yearly Means Plot of All Share Index**

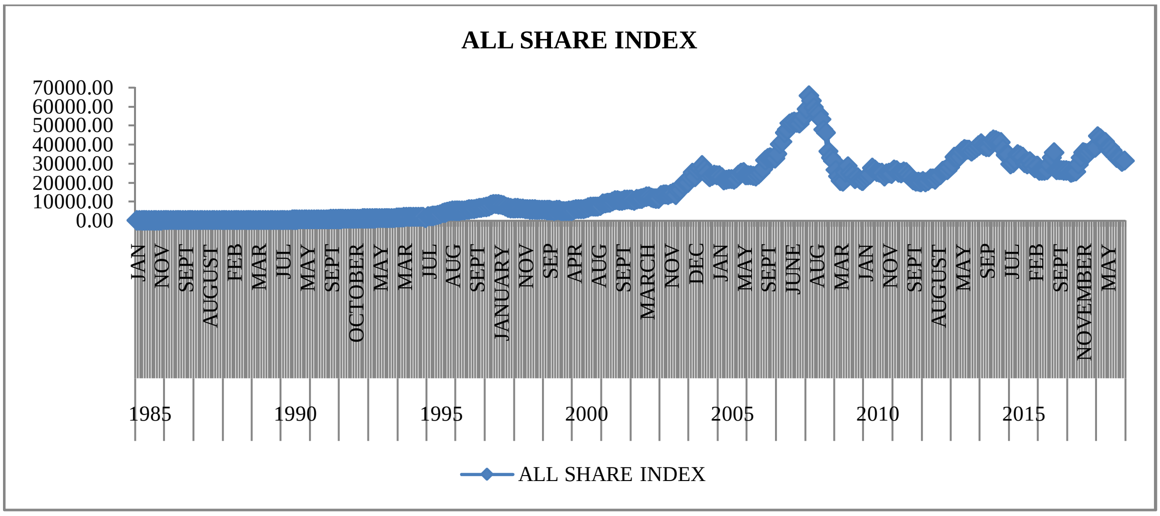


**Figure 4.3.Series Plot of All Share Index**

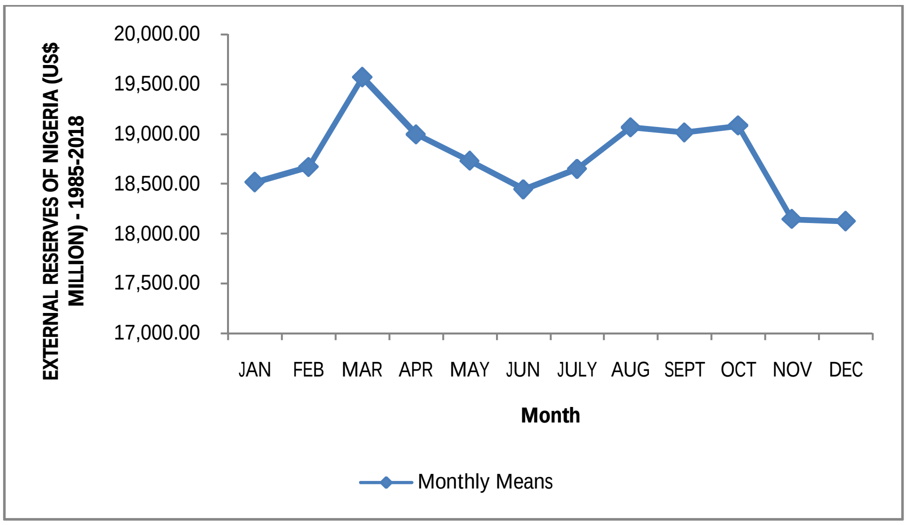


**Figure 4.4.Monthly Means Plot of External Reserves**

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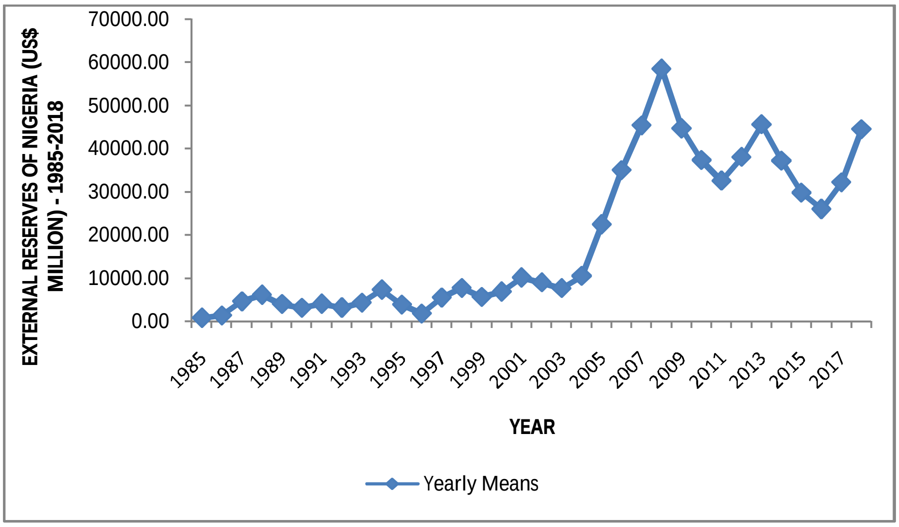


**Figure 4.5.Yearly Means Plot of External Reserves**

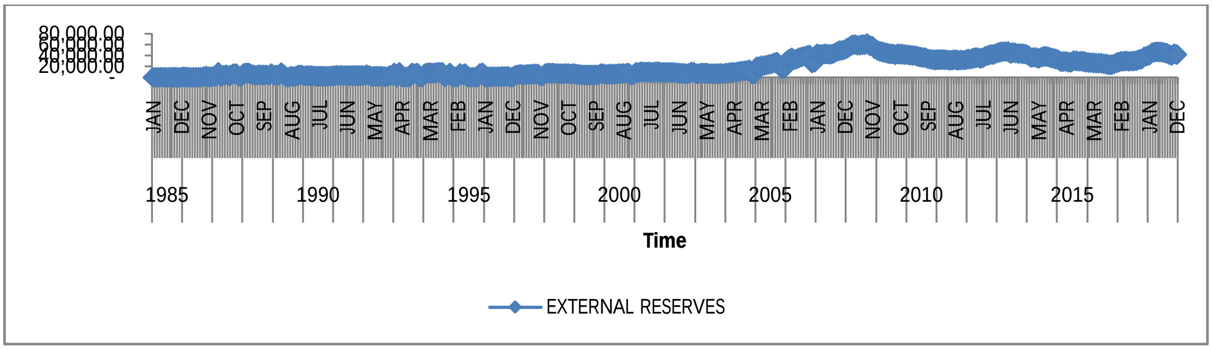


**Figure 4.6.Series Plot of External Reserves**

## 4.2 Stationary of the Data Sets and ARIMA Model Building

The plots in Figures 4.7 and 4.8 are the first difference series of ASI and ER respectively. The plots show a sine wave pattern in nature with mean zero and constants variance. The series is now stationary after the first difference (or behaves much better).

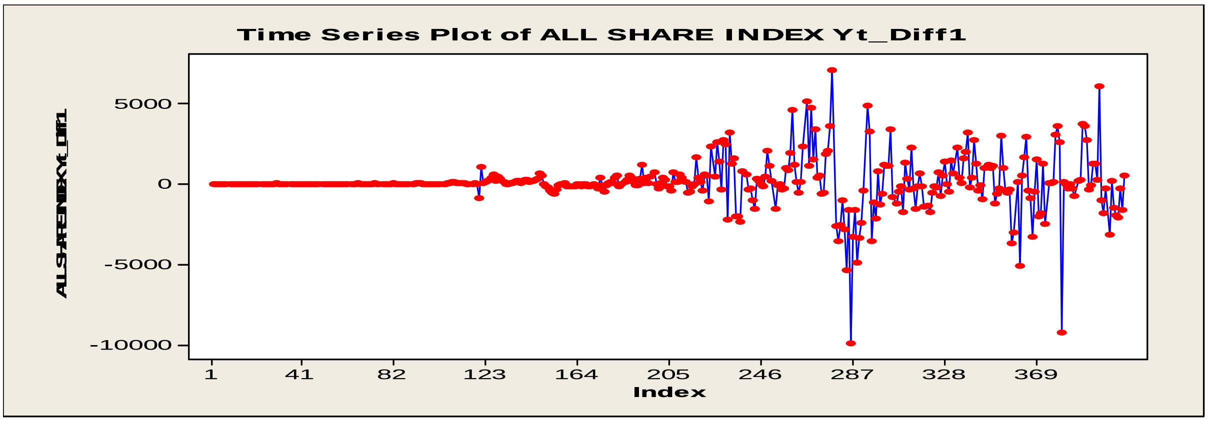


**Figure 4.7.First Difference of the All Share Index series**

 

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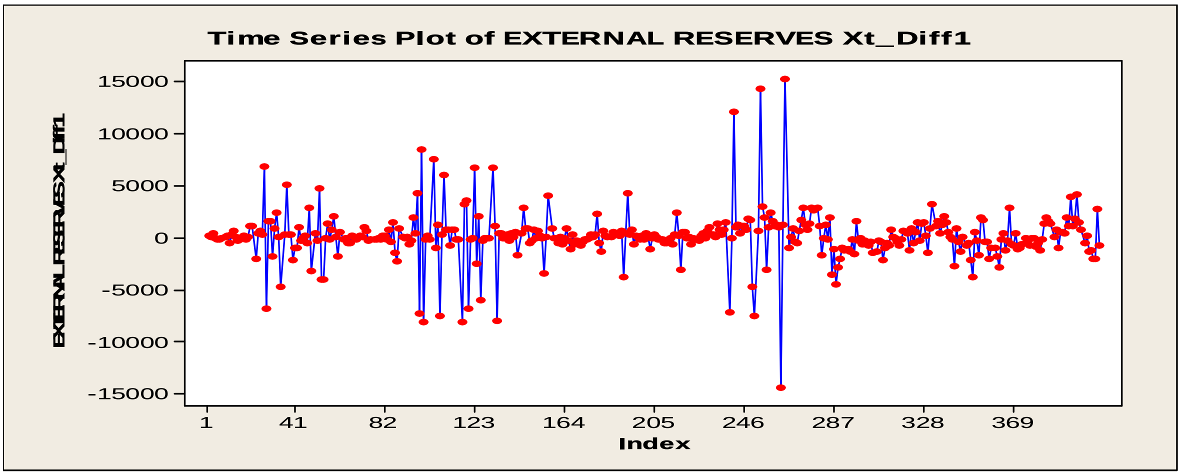


**Figure 4.8.First Difference of ER series**

## Parameters Estimates and ARIMA Model Identification

The ACF and PACF plots of the difference series for both ASI and ER series are shown in Figures 4.9 and 4.10 for ASI and Figure 4.11 and 4.12 for ER respectively. The ACF plot in Figure 4.9 shows spikes at lags 1 and 2 indicating AR(p) process (where p =1 or p =2). The PACF plots in Figure 4.10 shows a cut off at lag 2 and indicating MA(q), where q=1 or q=2.

However, various ARIMA(p, q) models were fitted to the All Share Index series with respective residuals as white noise and it is summarized in Table 4.1. The model selection criteria used to select the best model amongst models was AIC and BIC and also detailed out in Table 4.1. Hence, ARIMA (1,1,1) was identified for the ASI series in Table 4.1 and was used to forecasts for 2019.

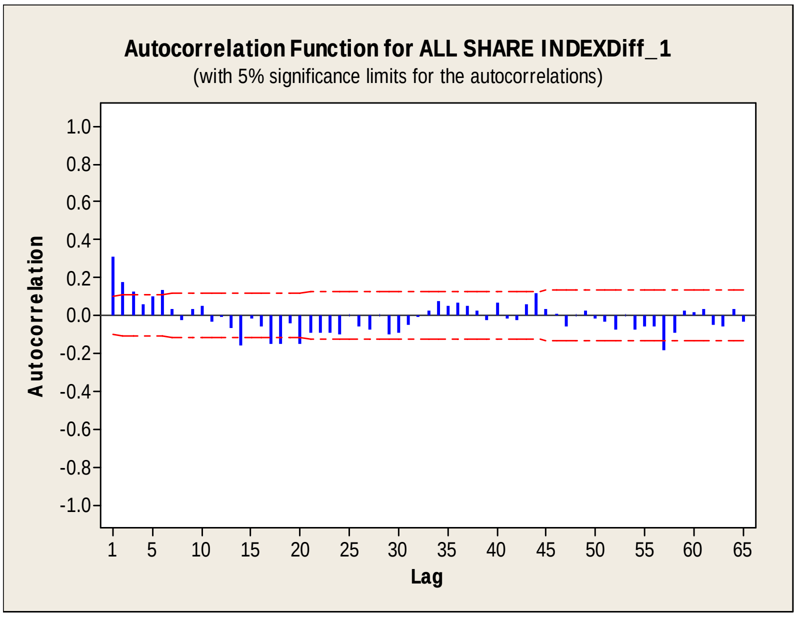


**Figure 4.9.ACF for First Difference ASI Series**

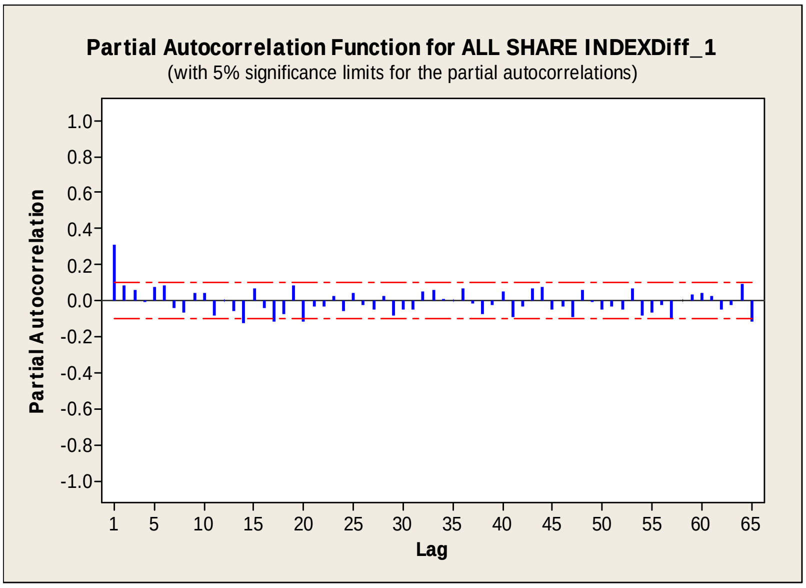


**Figure 4.10.PACF for First Difference ASI Series**

Similarly, in the ACF plot in Figure 4.11 for ER, spikes dies down extremely slowly indicating the AR(p) process (where p =1 or p =2). Also, PACF plots for ER spikes are close to white noise except lags 1 or 2 cut on, which is also an indication of MA(q), where q=1 or q=2. Also, the early lags indicated that these series have a seasonal variation of order 12. Then, various SARIMA(P, D, Q) models of order 12 were fitted to the ER series with respective residuals as white noise and its summarized in Table 4.2. The model selection criteria used to select the best model amongst models is AIC and BIC is also detailed in Table 4.2. Hence, ARIMA (0, 1, 0)(0, 0, $1)_{12}$ was identified for ER series in Table 4.2 and was used to forecasts for 2019.

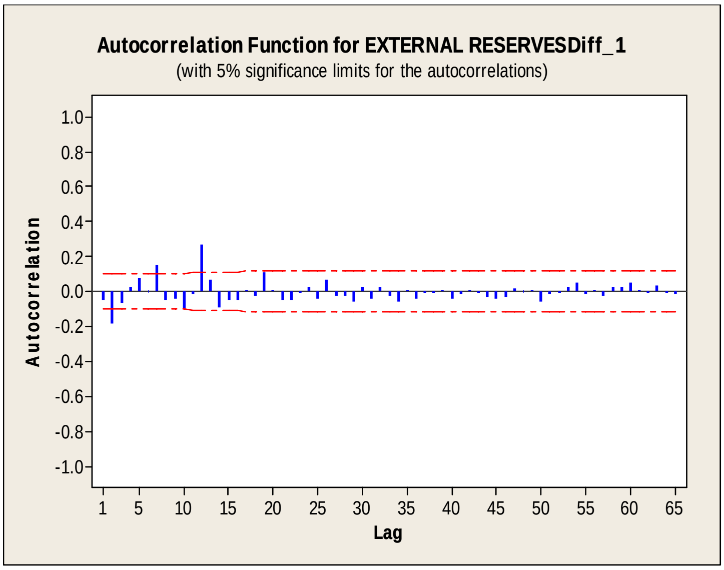


**Figure 4.11.ACF for First Difference ER Series**

 

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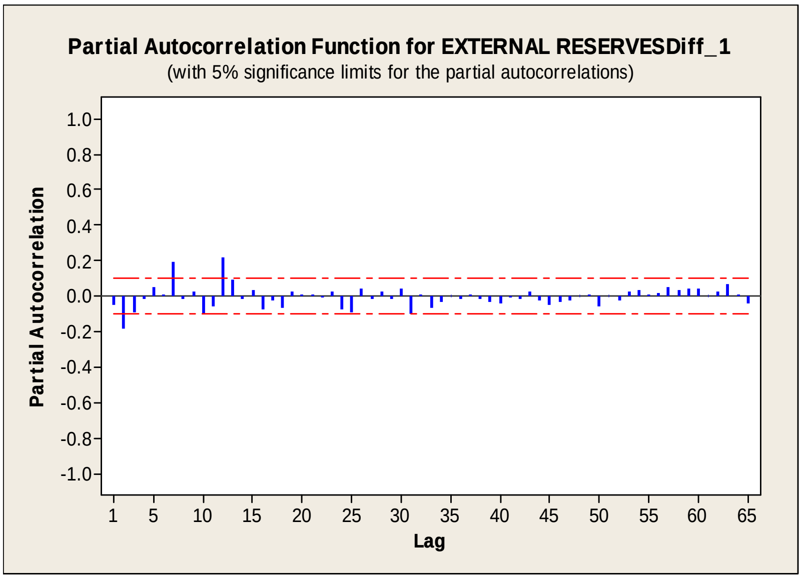


**Figure 4.12.PACF for First Difference ER Series**

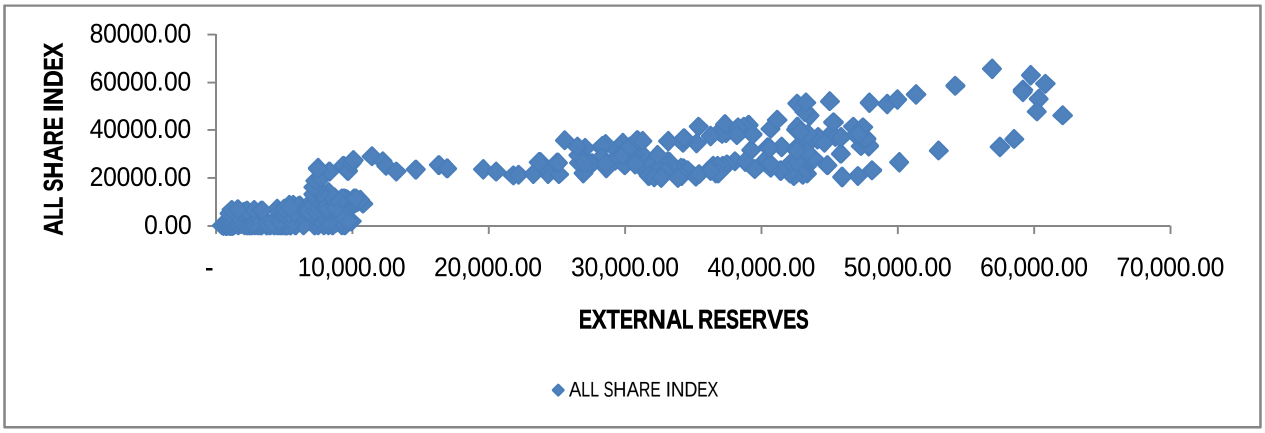


**Figure 4.13.Bivariate Plot of ASI on ER**

Figure 4.13 shows the Bivariate plot of ASI in ER. The plot indicates that there is a correlation between the two series. Also, Figure 4.14 shows the Bivariate plot of ER on ASI. The plot also indicates a correlation between the two series. Thus, there is a strong relationship between ASI and ER series.

## 4.3 Identification of VAR Models and Its Parameters Estimates

The two series plotted in Figure 4.13 and 4.14 are correlated, also from the correlation matrix, it is obvious the ASI and ER series are significantly correlated, using Minitab 17 statistical software (Pearson correlation of ASI and ER = 0.909 (p-value = 0.000). Hence, the series can be considered as a bivariate series using VAR Model.

**Table 4.1. and Table 4.2 (end of the paper)**

 

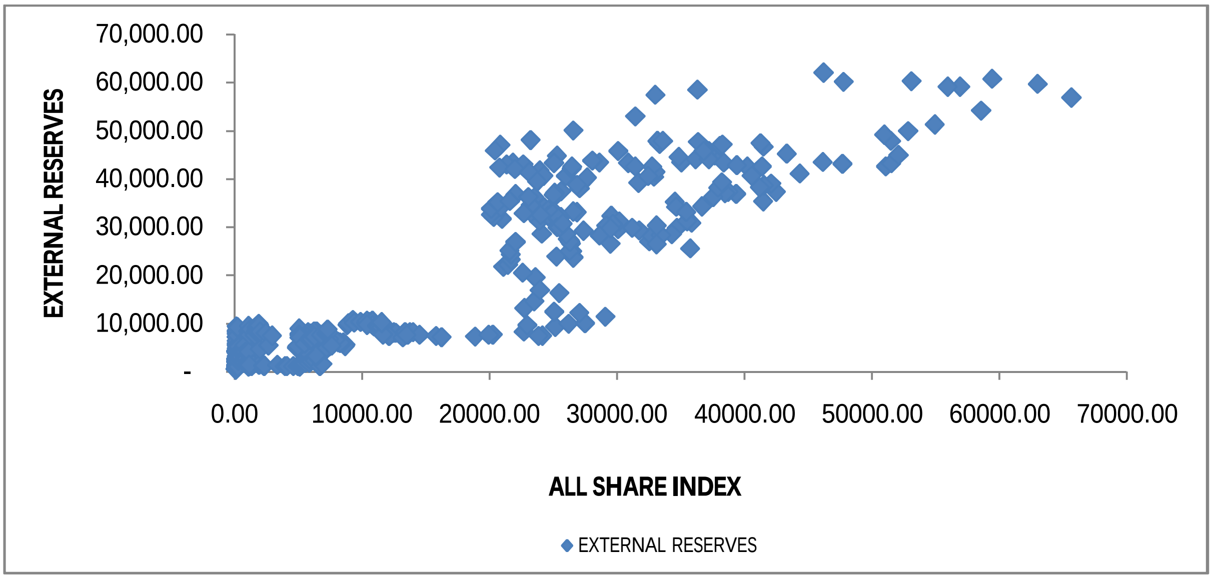


**Figure 4.14.Bivariate Plot of External Reserves on All Share Index**

### 4.3.1 ASI against ER Series Parameters Estimates and VAR Model Identification

### A) VAR Model: Lag Length Selection

The lag length for the VAR(p) model may be determined using model selection criteria. The general approach is to fit VAR(p) models with order P = $P_0$, $P_1$,...,$P_{max,}$ and choose the value of P which minimizes some model selection criteria. The comparison made between the two series summarized (with maximum lag order 4) in Table 4.3.

**Table 4.3.Max Lag Length selection of All Share Index on External Reserves Series**

| Lags | Loglik | p(LR) | AIC | BIC | HQC |
|---|---|---|---|---|---|
| 1 | -3531.91 | | 17.50449 | 17.54411 | 17.52017 |
| 2 | -3509.40 | 0.00000 | 17.39801 | 17.44753* | 17.41761 |
| 3 | -3506.91 | 0.02574 | 17.39065 | 17.45008 | 17.414173* |
| 4 | -3505.34 | 0.07657 | 17.38784* | 17.45717 | 17.41528 |

The AIC selection criteria suggested VAR(4), while BIC and HQC selection criteria suggested VAR(2) and VAR(3) respectively. However, VAR(1) to VAR(4) models were fitted and using information criteria, VAR(3) model was the best model for ASI on ER series because it has the lowest model selection criteria and highest $R^2$ and $R^2$ adjusted (99.12% and 99.11%). Also, three parameters of the VAR(3) model were significant at 1%, 5%, and 10%.

### B): VAR Model Estimation Parameters

Considering the estimates of the models identified (VAR(3)), The general vector autoregressive model of order 2 is represented as:

$$Y_{1t} = \mu_1 + \phi_{11} Y_{1t-1} + \phi_{12} Y_{1t-2} + ... + \phi_{1p} Y_{1t-p} + \phi_{21} Y_{2t-1} + \phi_{22} Y_{2t-2} + ... + \phi_{2p} Y_{2t-p} + \varepsilon_{1t}$$

 

*where* $Y_{1t}$ is the All Share Index series and $Y_{2t}$ is the External Reserves Series

**Table 4.4.VAR(3) model Parameter Estimates of All Share Index on External Reserves Series**

| **Variable** | **Co-efficient** | **Std-error** | **t-ratio (p-value)** | **Remark** |
|---|---|---|---|---|
| Constant | 161.972 | 109.963 | 1.4730 (0.1415) | Not significant |
| $Y_{1t-1}$ | 1.2801 | 0.0503 | 25.4302 (0.0000***) | **Significant** |
| $Y_{1t-2}$ | -0.1969 | 0.0816 | -2.4141 (0.0162**) | **Significant** |
| $Y_{1t-3}$ | -0.0928 | 0.0519 | -1.7884 (0.0745*) | **Significant** |
| $Y_{2t-1}$ | -0.0005 | 0.0310 | -0.0164 (0.9869) | Not significant |
| $Y_{2t-2}$ | 0.0031 | 0.0302 | 0.1012 (0.9194) | Not significant |

**Footnote:** ***-sig. at 1%,**-sig. at 5%,*-sig. at 10%,

Table 4.4 revealed that the value of ASI on ER at any given time using the VAR(3) model is determined by the equation below;

$$Y_{1t} = 1.2801\, Y_{1t-1} - 0.1969\, Y_{1t-2} - 0.0928\, Y_{1t-3} + \varepsilon_{1t} \qquad (4.1)$$

**(C): Forecasts**

The VAR model identified was used to generate forecasts. Hence, the vector autoregressive model is represented as [VAR(3)];

$$Y_{1t} = 1.2801\, Y_{1t-1} - 0.1969\, Y_{1t-2} - 0.0928\, Y_{1t-3} + \varepsilon_{1t}$$

$$Y_{2t} = 0.87401\, Y_{2t-1} - 0.1316\, Y_{2t-2} + 0.1853\, Y_{2t-3} + 0.1706\, Y_{1t-1} + \varepsilon_{1t}$$

### 4.3.2 ER against ASI Series Parameters Estimates and VAR Model Identification

**A): VAR Model: Lag Length Selection**

The general approach is to fit VAR(p) models with order P = $P_0$, $P_1$,...,$P_{max,}$ and choose the value of P which minimizes some model selection criteria. The summary of the lag length selection is given in Table 4.5

**Table 4.5.Max Lag Length selection of External Reserves Series onAll Share Index**

| **Lags** | **loglik** | **p(LR)** | **AIC** | **BIC** | **HQC** |
|---|---|---|---|---|---|
| 1 | -3707.72 | | 18.36990 | 18.39962 | 18.38167 |
| 2 | -3707.36 | 0.39453 | 18.37306 | 18.41268 | 18.38874 |
| 3 | -3700.16 | 0.00015 | 18.32392 | 18.39191* | 18.36199 |
| 4 | -3697.92 | 0.03443 | 18.33627* | 18.39570 | 18.35980* |

 

The AIC selection criteria suggested VAR(4), while BIC and HQC selection criteria suggested VAR(3) and VAR(4) respectively. However, VAR(1) to VAR(4) models were fitted. A comparison of the information criteria of the four VAR models fitted shows that the VAR(3) model was the best model for ER on ASI series because it has the lowest model selection criteria and highest $R^2$ and $R^2$ adjusted (98.21% and 98.18%). Also, three parameters of the VAR(3) model were significant at 1% and 5%.

### B): VAR Model Estimation Parameters

**Table 4.6.VAR(3) model Parameter Estimates of External Reserves on ASI**

| Variable | Co-efficient | Std-error | t-ratio (p-value) | Remark |
|---|---|---|---|---|
| Constant | 156.757 | 172.02 | 0.9113 (0.3627) | Not significant |
| $Y_{2t-1}$ | 0.8740 | 0.0494 | 17.6702 (0.0000***) | **Significant** |
| $Y_{2t-2}$ | -0.1316 | 0.0648 | -1.9980 (0.0464**) | **Significant** |
| $Y_{2t-3}$ | 0.1852 | 0.0482 | 3.8427 (0.0000***) | **Significant** |
| $Y_{1t-1}$ | 0.1706 | 0.0770 | 2.2162 (0.0162**) | **Significant** |
| $Y_{1t-2}$ | -0.0893 | 0.0782 | -1.1264 (0.9194) | Not significant |

**Footnote:** ***-sig. at 1%,**-sig. at 5%,*-sig. at 10%,

Table 4.6 reveals that the value of ER on ASI at any given time using the VAR(3) model is determined by the equation below;

$$Y_{2t} = 0.87401\,Y_{2t-1} - 0.1316\,Y_{2t-2} + 0.1853\,Y_{2t-3} + 0.1706\,Y_{1t-1} + \varepsilon_{1t} \quad (4.2)$$

### (C): Forecasts

The VAR model identified was used to generate the forecasts. Hence, the vector autoregressive model is represented as [VAR(3)];

$$Y_{1t} = 1.2801\,Y_{1t-1} - 0.1969\,Y_{1t-2} - 0.0928\,Y_{1t-3} + \varepsilon_{1t}$$

$$Y_{2t} = 0.87401\,Y_{2t-1} - 0.1316\,Y_{2t-2} + 0.1853\,Y_{2t-3} + 0.1706\,Y_{1t-1} + \varepsilon_{1t}$$

### 4.4 Models Forecasts Comparison (VAR and ARIMA Model Identified Forecasts)

The estimated errors were used to compare the two models forecasts This is done by subtracting the estimated forecast values ($F_i$) from the original values or [actual values ($A_i$)] to obtain the estimated errors. The VAR and ARIMA model identified forecasts in Table 4.6 is compared to determine the suitable model between the two models for forecasting Nigeria ASI and ER, using forecast accuracy measures.

International Journal of Transformation in Applied Mathematics & Statistics
Vol. 3, Issue 2 – 2020
ISSN: 2581-7620

**Table 4.7.Forecast Accuracy Measures of ARIMA and VAR model Comparison**

| Variable | FAM | ARIMA model | Variable | VAR Model |
|---|---|---|---|---|
| All Share Index (ASI) ARIMA(1,1,1) | MAE | 740.18 | ASI against ER VAR(3) | 790.67 |
| | MSE | 2054030.40 | | 2045618.36 |
| | MAPE | 1.002% | | 0.214% |
| External Reserves (ER) ARIMA(0,1,0)(0,,0,1)$1_{12}$ | MAE | 1340.33 | ER against ASI VAR(3) | 1376.92 |
| | MSE | 5606178.81 | | 5298057.42 |
| | MAPE | 1.003% | | 0.261% |

From Table 4.7, the bivariate model is better to predict the two series than the univariate model, using accuracy measures (i.e. MAPE and MSE).

## 5 Conclusion

The methods used are plots, descriptive statistics, stationarity; ARIMA models identification, and the VAR model's identification. The variables were examined in terms of correlation relationships, trend component, and seasonality effect if present in the data. The two series plots (ASI and ER) were compared and it was noticed that there is a similar behaviour between the two series which shows an upward trend component. The series was difference once to obtain a stationary series. A suitable ARIMA model was identified for the two series (univariate model). In fitting a multivariate time series model called VAR Model, the two series plotted were correlated and their correlation matrix was computed. The ASI and ER series are significantly correlated with 0.909 (p-value = 0.000). Thus, the series can be considered as a bivariate series, and the bivariate analysis was done using a VAR Model. The four VAR models fitted show that the VAR(3) model was the best model for ASI against ER and ER against ASI using information criteria. Also, the three parameters of the VAR(3) model were significant at 1%, 5%, and 10%.

The univariate and bivariate model forecasts were compared and the result shows that the bivariate model is better to predict the two series than the univariate model from the result of forecast accuracy measures (i.e. MAPE and MSE). It seems reasonable to conclude that there is a significant relationship between the ASI and ER series.

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**Table 4.1.Identification of ARIMA Model for ASI**

| | AR(p) Estimates | | | MA(q) Estimates | | | Modified (Ljung-Box) statistic | | Box-Pierce Chi-Square | | RSS ($\sigma^2$) | AIC | Rank | BIC | Rank | Average Rank |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ARIMA Models | $\phi_1$ | $\phi_2$ | $\phi_3$ | $\theta_1$ | $\theta_2$ | $\theta_3$ | k=12 | k=24 | k=36 | k=48 | | | | | | |
| ARIMA(1,1,0) | 0.3135 (0.000***) | | | | | | 15.3 (11) | 54.4 (23) | 63.8 (35) | 85.5 (47) | 846536152 | 5936.52 | 6 | 5940.53 | 1 | 3.5 |
| ARIMA(2,1,0) | 0.2867 (0.000***) | 0.0853 (0.086*) | | | | | 13.5 (10) | 50.2 (22) | 64.1 (34) | 81.8 (46) | 840401081 | 5935.55 | 4 | 5943.58 | 3 | 3.5 |
| ARIMA(3,1,0) | 0.2820 (0.000***) | 0.0692 (0.000***) | 0.0560 (0.000***) | | | | 11.3 (9) | 44.0 (21) | 57.5 (33) | 75.4 (45) | 837779274 | 5936.28 | 5 | 5948.31 | 8 | 6.5 |
| ARIMA(0,1,1) | | | | -0.2665 (0.000***) | | | 23.1 (11) | 66.3 (23) | 81.3 (35) | 96.5 (47) | 862351489 | 5944.07 | 9 | 5948.09 | 7 | 8 |
| ARIMA(0,1,2) | | | | -0.2721 (0.000***) | -0.1236 (0.013**) | | 18.7 (10) | 60.9 (22) | 75.8 (34) | 92.3 (46) | 849257070 | 5939.83 | 8 | 5947.85 | 6 | 7 |
| ARIMA(0,1,3) | | ;/. | | -0.2894 (0.000***) | -0.1534 (0.003***) | -0.993 (0.0046***) | 12.0 (9) | 48.7 (21) | 61.8 (33) | 80.2 (45) | 839173292 | 5936.96 | 7 | 5948.99 | 9 | 8 |
| **ARIMA(1,1,1)** | **0.6234 (0.000***)** | | | **0.3523 (0.009***)** | | | **12.1 (10)** | **45.4 (22)** | **59.2 (34)** | **77.1 (46)** | **838045172** | **5934.41** | **1** | **5942.43** | **2** | **1.5** |
| ARIMA(2,1,1) | 0.9148 (0.001***) | -0.1107 (0.357) | | 0.6363 (0.022**) | | | 10.5 (9) | 42.6 (21) | 56.8 (33) | 74.3 (45) | 836117641 | 5935.47 | 2 | 5947.50 | 4 | 3 |
| ARIMA(1,1,2) | 0.7615 (0.000***) | | | 0.4831 (0.001***) | 0.0742 (0.324) | | 10.3 (9) | 41.9 (21) | 56.2 (33) | 73.6 (46) | 836163503 | 5935.49 | 3 | 5947.53 | 5 | 4 |

**FOOTNOTE:** ***-sig. at 1%, **-sig. at 5%,*-sig. at 10%

**Table 4.2.Identification of ARIMA Model for ER**

| | SAR(p) Estimates | | | SMA(q) Estimates | | | Modified (Ljung-Box) Box-Pierce Chi-Square statistic | | | | RSS | AIC | Rank | BIC | Rank |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| SARIMA of Order 12 | $\phi_1$ | $\phi_2$ | $\phi_3$ | $\theta_1$ | $\theta_2$ | $\theta_3$ | K=12 | K=24 | K=36 | K=48 | | | | | |
| **ARIMA(0,1,0)(1,0,0)$_{12}$** | 0.2724 (0.000***) | | | | | | 24.4 (11) | 38.9 (23) | 53.1 (35) | 56.3 (47) | 2205778197 | 6327.26 | 2 | 6331.27 | 2 |
| **ARIMA(0,1,0)(2,0,0)$_{12}$** | 0.2859(0.000***) | -0.0506 (0.3150) | | | | | 24.1 (10) | 39.3 (22) | 52.0 (34) | 55.8 (46) | 2200330467 | 6328.25 | 4 | 6336.27 | 4 |
| **ARIMA(0,1,0)(0,0,1)$_{12}$** | | | | **-0.2701 (0.000***)** | | | **24.6 (11)** | **40.7 (23)** | **53.0 (35)** | **56.9 (47)** | **2205726430** | **6327.25** | **1** | **6331.26** | **1** |
| **ARIMA(0,1,0)(0,0,2)$_{12}$** | | | | -0.2866 (0.000***) | -0.0522 (0.3010) | | 24.0 (10) | 38.8 (22) | 52.0 (34) | 55.6 (46) | 2200289324 | 6328.24 | 3 | 6336.26 | 3 |
| **ARIMA(1,1,0)(0,0,0)$_{12}$** | -0.0491 (0.323) | | | | | | 66.1 (11) | 82.8 (23) | 90.6 (35) | 93.9 (47) | 2373454854 | 6357.15 | 5 | 6361.16 | 5 |

**FOOTNOTE:** ***-sig. at 1%, **-sig. at 5%,*-sig. at 10%,