International Journal of Transformation in Applied Mathematics & Statistics
Vol. 3, Issue 1 – 2020
ISSN: 2581-7620

Cordial labeling of Butterfly and Enhanced Butterfly Networks

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Abstract

Let G = (V, E) is a graph with vertex set V and edge set E.A vertex labeling $f: V \to \{0,1\}$ induces an edge labeling $f^* E \to \{0,1\}$defined by$f^*(x,y) = |f(x) - f(y)|$. For $i \in \{0,1\}$ let $vf(i)$ and $ef(i)$ be the number of vertices v and edges e with $f(v) = i$ and$f^*(e) = i$, respectively. A graph G is cordial if there exists a vertex labeling f such that$|vf(0) - vf(1)| \leq 1$ and$|ef(0) - ef(1)| \leq 1$. The cordial labeling of butterfly and enhanced butterfly networks are studied in this paper.

Keywords: Cordial labeling, Butterfly Network and Enhanced Butterfly Network.

Introduction

Cordial labeling is one of the predominant labeling in graph theory. Inter connection networks are widely used in chip designing. Butterfly (BF) graph is a inter connection network. In 1987, Cahit presented the cordial labeling idea as weaker variant of elegant and harmonious labeling. A binary vertex labeling of a graph G is called a cordial labeling if $|vf(0) - yf(1)| \leq 1$ and$|ef(0) - ef(1) \leq 1|$. A graph which admits cordial labeling is called a cordial graph.

Definition

The n dimensional butterfly network, denoted by BF(n), has a vertex set$V = \{(x,i); x \in V(Q_n), o \leq i \leq n\}$. Two vertices (x;i) and (y;j) are linked by an edge in BF(n) if and only if j=i+1 and either

(i) x=y or
(ii) x differs from y in precisely the j^{th} bit.

For x =y, the edge is said to be straight edge, otherwise the edge is a cross edge. For fixed i the vertex (x;i) is a vertex on level i.

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- Butterfly networks are smaller in diameter than other topologies such as a linear array, loop, and 2D mesh. This means a message sent from one processor in a butterfly network will hit its destination in a smaller number of network hops. Butterfly networks have an improved bisection bandwidth relative to other topologies.
- It means that a larger number of connections need to be broken in the butterfly network in order to avoid global communication.
- It has greater range of computers.

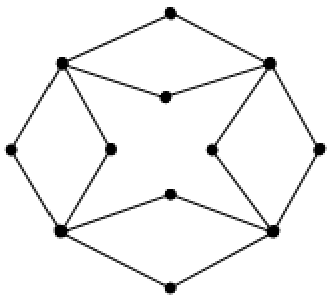

Figure 1.BF(2)

Theorem: 1

Butterfly network is cordial labeling graph.

Proof:

Consider a BF(2) and label the vertices as follows:

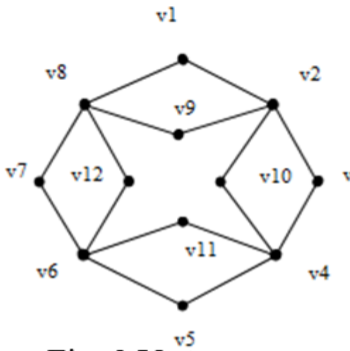

Fig. 2 Vertex labeling of BF (2)

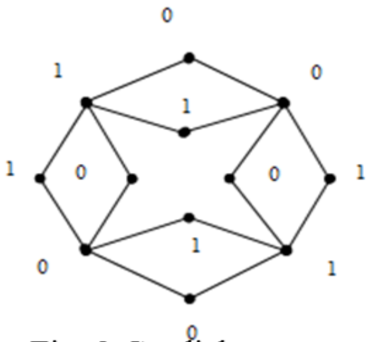

Fig. 3 Cordial vertex labeling of BF (2)

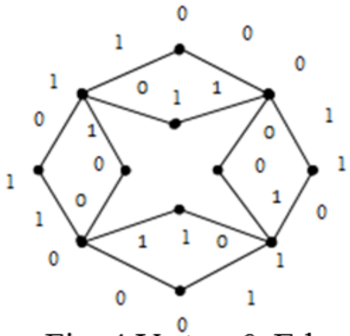

Fig. 4 Vertex & Edge cordial labeling of BF (2)

$$v_i = \begin{cases} \left\lceil \frac{i}{2} \right\rceil + 1 \; mod \; 2, 1 \le i \le 8 \\ i \; mod \; 2, 9 \le i \le 12 \end{cases}$$

$\Rightarrow vf(0) = vf(1) = 6$

Therefore, $|vf(0) - vf(1)| = 0 \le 1$

$\Rightarrow ef(0) = ef(1) = 8$

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Therefore, $|ef(0) - ef(1)| = 0 \leq 1$

For BF (3),

BF (3) contains of two copies of BF (2) and four copies BF (1) connecting the two BF (2).

Considering the same labeling of BF (2) as one copy and taking the compliment of the BF (2); label the second BF (2).

The mapping as follows:

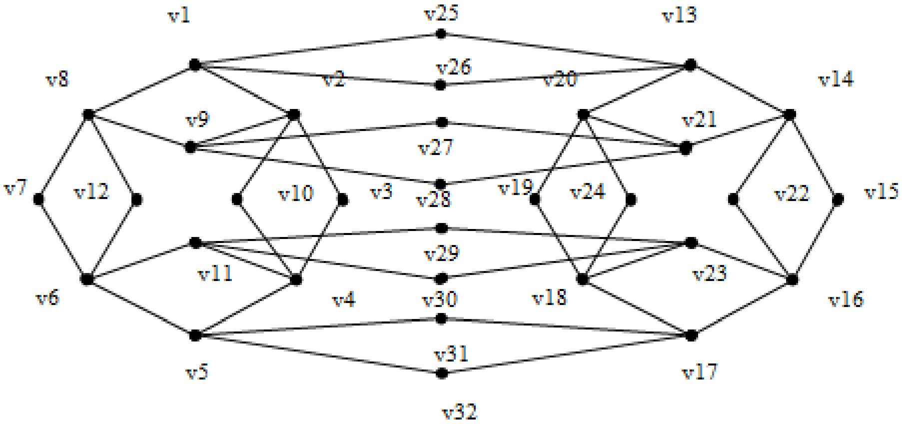

Figure 5.Vertex labeling of BF (3)

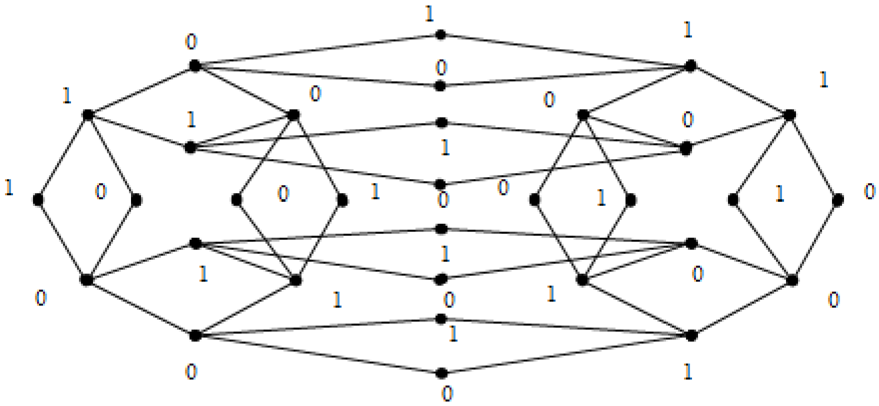

Figure 6.Vertex Cordial labeling of BF (3)

$$v_i = \begin{cases} \left\lceil \frac{i}{2} \right\rceil + 1\ mod\ 2, 1 \leq i \leq 8 \\ i\ mod\ 2, 9 \leq i \leq 12 \\ \left\lceil \frac{i}{2} \right\rceil\ mod\ 2, 13 \leq i \leq 20 \\ i + 1\ mod\ 2, 21 \leq i \leq 24 \\ i\ mod\ 2, 25 \leq i \leq 32 \end{cases}$$

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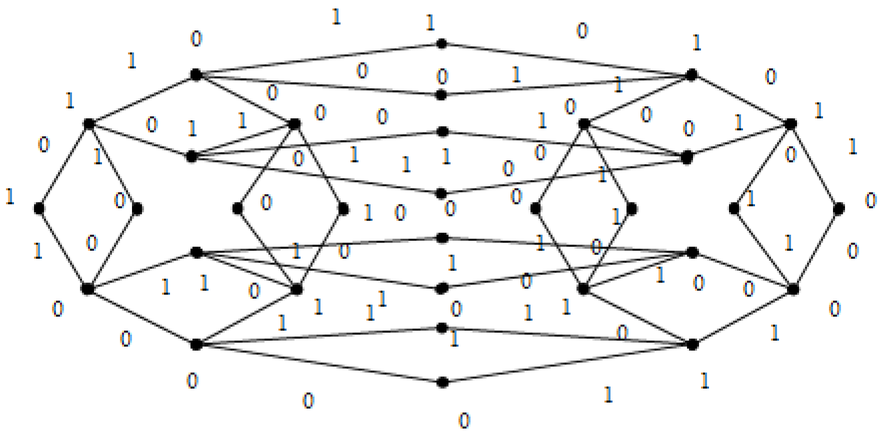

Figure 7.Vertex & Edge Cordial labeling BF (3)

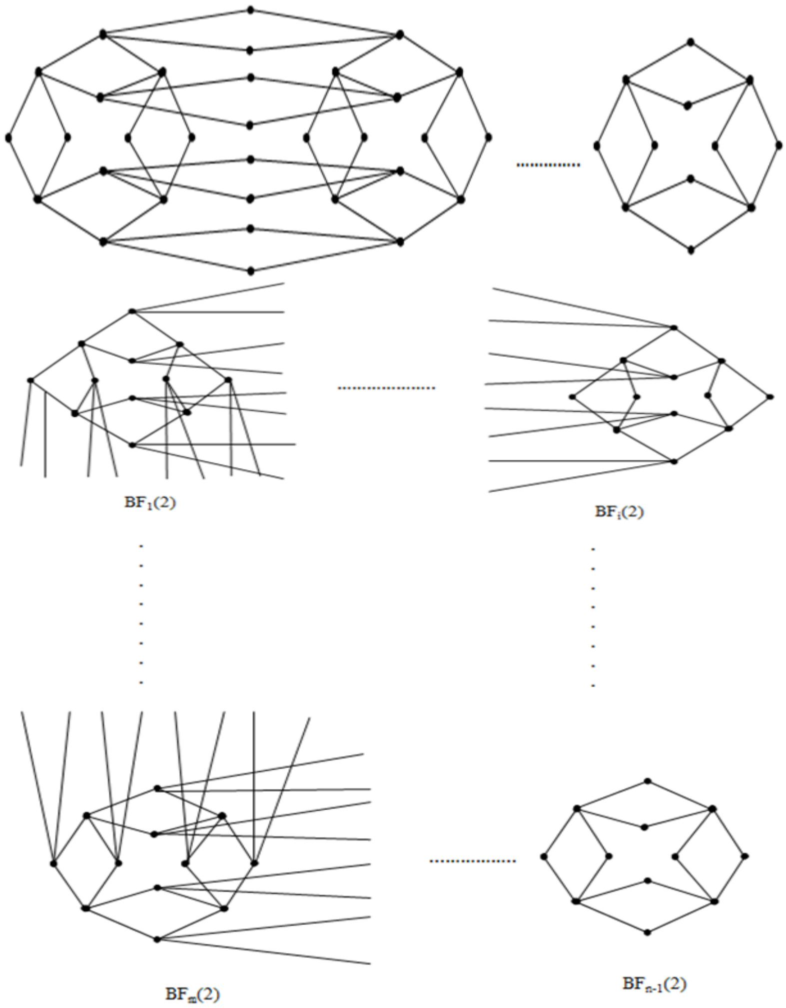

Figure 8.BF (n)

$vf(0) = vf(1) = 16$

Therefore, $|vf(0) - vf(1)| = 0 \leq 1$

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$ef(0) = ef(1) = 24$

Therefore, $|ef(0) - ef(1)| = 0 \leq 1$

Hence BF (3) possesses cordial labeling.

In general for BF (n), there are n-1 copies of BF (2) connected by 2^{n-1} copies of BF (1).

$vf(0) = vf(1) = (n+1)2^{n-1}$

Therefore, $|vf(0) - vf(1)| = 0 \leq 1$

$ef(0) = ef(1) = n2^{n}$

Therefore, $|ef(0) - ef(1)| = 0 \leq 1$

Enhanced Butterfly Network (EBF)

Definition

Consider the r-dimensional butterfly network BF(r). Place a new vertex in each 4-cycle of BF(r) and join this vertex to the four vertices of the 4-cycle. The resulting graph is called an enhanced butterfly network BF(r). This network has 2^{r-1} (3r+2) vertices and r × 2^{r+2} edges. Additional vertices are given the labels ($x_1x_2\ldots\ldots x_k$, $x_1'x_2'\ldots\ldots x_k'$) to the vertices with labels ($x_1x_2\ldots\ldots x_k$, i), ($x_1'x_2'\ldots\ldots x_k'$, i+1), ($x_1'x_2'\ldots\ldots x_k'$, i), ($x_1x_2\ldots\ldots x_k$, i+1), $0 \leq i \leq r$.

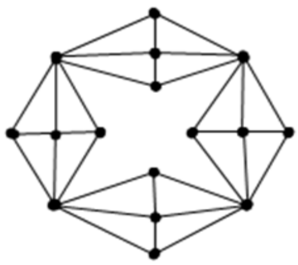

Fig. 9 EBF (2)

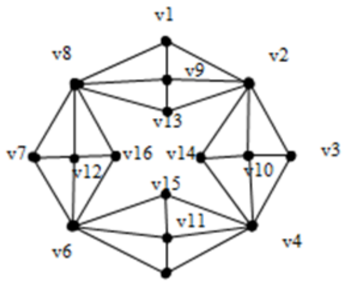

Fig. 10 Vertex labeling of EBF (2)

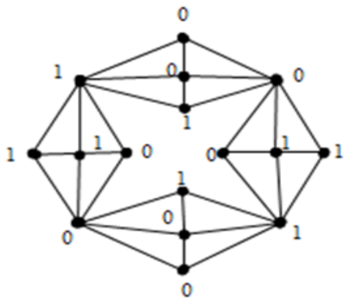

Fig. 11 Vertex Cordial labeling of EBF (2)

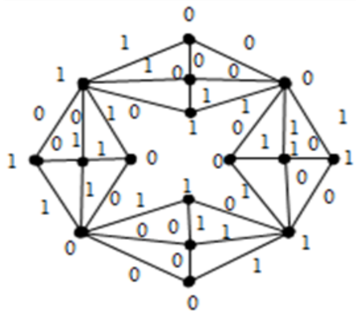

Fig. 12 Vertex & Edge Cordial labeling of EBF (2)

Theorem: 2

The EBF Network (n) is cordial labeling.

Proof

The EBF(2) can be vertex labeled as follows:

$$v_i = \begin{cases} \left\lceil \frac{i}{2} \right\rceil + 1\ mod\ 2, 1 \leq i \leq 8 \\ i\ mod\ 2, 9 \leq i \leq 12 \\ i + 1\ mod\ 2, 13 \leq i \leq 16 \end{cases}$$

$vf(0) = vf(1) = 8$

Therefore, $|vf(0) - vf(1)| = 0 \leq 1$

$$ef(0) = ef(1) = 14$$

Therefore, $|ef(0) - ef(1)| = 0 \leq 1$

Hence EBF (2) is cordial labeling.

For EBF (3),

Two copies of EBF (2) is connected by four copies of EBF (1).

The mapping of EBF (3) is as follows.The first copy of EBF (2) is the same from the previous case and the same copy is the complement of the first copy (ie). 0’s and 1’s are interchanged.

$$v_i = \begin{cases} \left\lceil \frac{i}{2} \right\rceil + 1\ mod\ 2, 1 \leq i \leq 8 \\ i\ mod\ 2, 9 \leq i \leq 12 \\ i + 1\ mod\ 2, 13 \leq i \leq 16 \\ \left\lceil \frac{i}{2} \right\rceil\ mod\ 2, 17 \leq i \leq 24 \\ i + 1\ mod\ 2, 25 \leq i \leq 28 \\ i\ mod\ 2, 29 \leq i \leq 32 \\ \left\lceil \frac{i}{2} \right\rceil + 1\ mod\ 2, 33 \leq i \leq 44 \end{cases}$$

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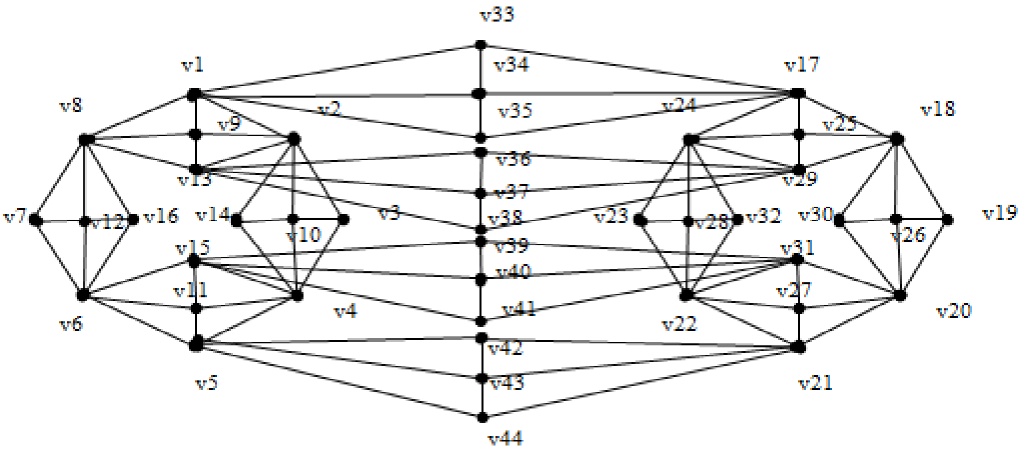

Figure 13.Vertex labeling of EBF (3)

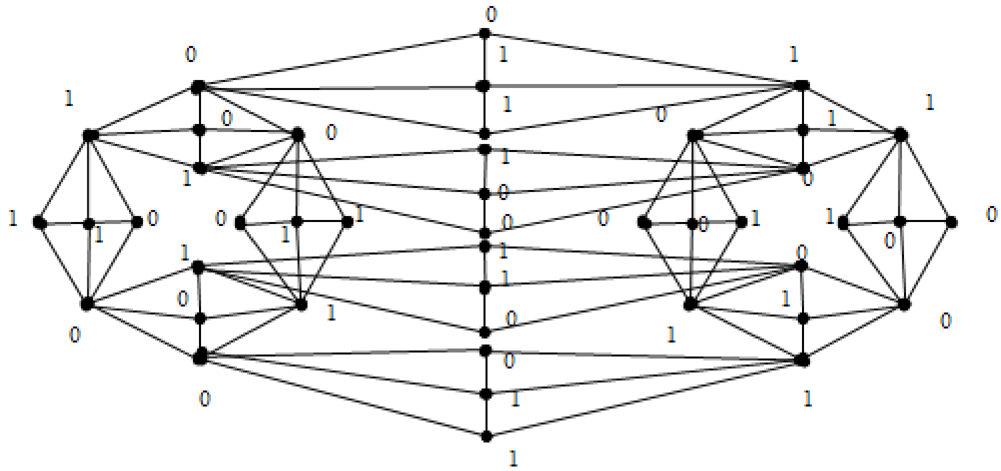

Figure 14.Cordial labeling of EBF(3)

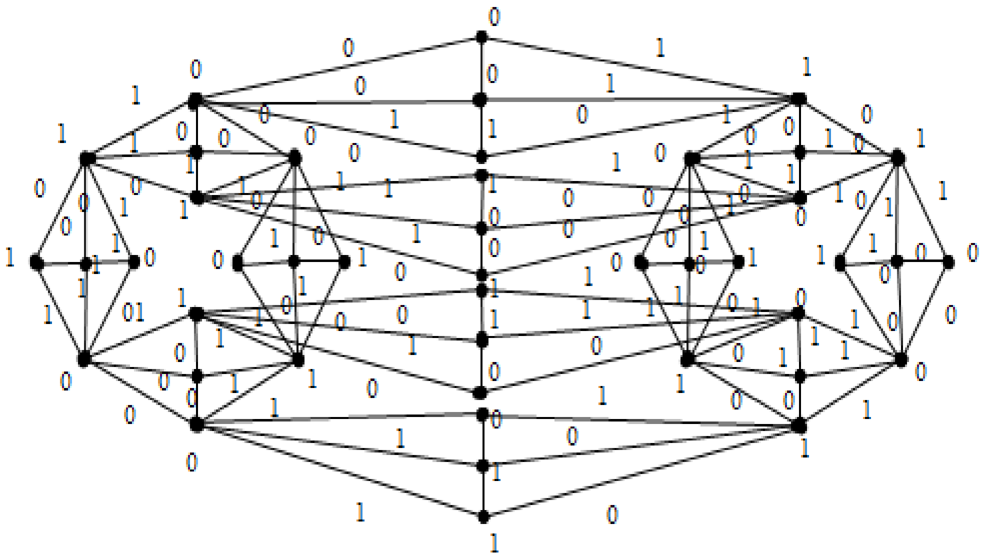

Figure 15.Vertex & Edge Cordial labeling of EBF (3)

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$vf(0) = vf(1) = 22$

Therefore,$|vf(0) - vf(1)| = 0 \leq 1$

$ef(0) = ef(1) = 44$

Therefore, $|ef(0) - ef(1)| = 0 \leq 1$

EBF (3) is cordial labeling.

In general for EBF (n),

The n-1 copies of EBF (2) in EBF (n) are labeled with first copy of EBF (2) as in previous case and the rest of the copies are taken compliment from the previous copies (ie)., EBF_1 (2), EBF_3(2) …… are taken the same as EBF_1 (2) and EBF_2 (2), EBF_4 (2),……… are taken as compliment of EBF_1 (2).

The n-dimensional EBF diagram was created by the 3-dimentional EBF diagram.

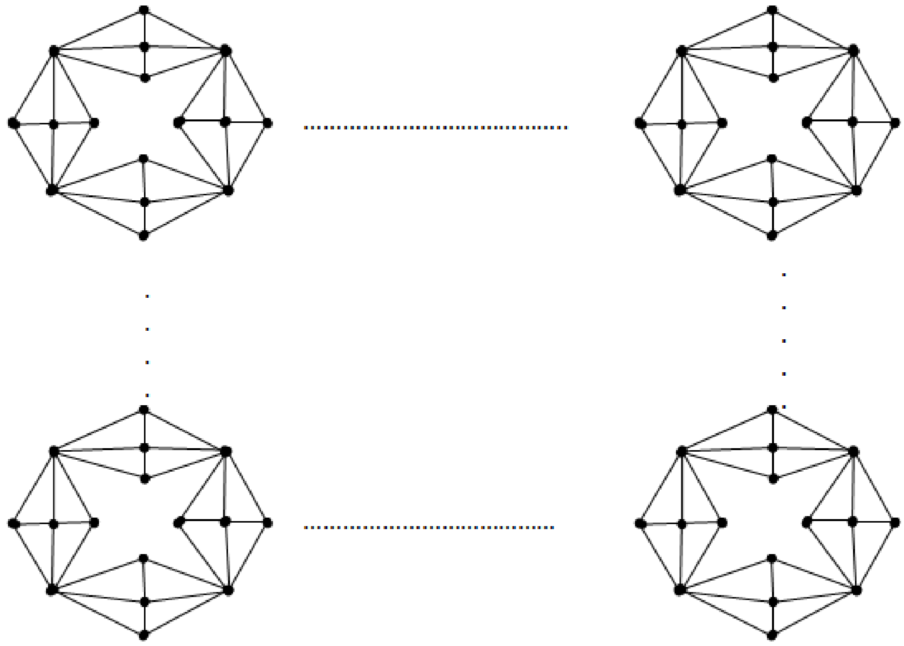

Figure 16.EBF (n)

$vf(0) = vf(1) = 2^{n-1}(3n + 2)$

Therefore, $|vf(0) - vf(1)| = 0 \leq 1$

$ef(0) = ef(1) = 0 \leq 1$

Therefore, $|ef(0) - ef(1)| = 0 \leq 1$

Therefore, EBF (n) is cordial labeling.

Conclusion

In this paper the cordial labeling of butterfly and enhanced butterfly network is studied. Cordial labeling can be applied for other inters connection networks in graph theory.

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