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Folding of Symmetrical Planar Shapes without Breaking Symmetry

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Abstract

In this paper I focused on folding symmetrical shapes like square rectangle circle and other polygons about their line of symmetries. Then I classify these shapes in terms of foldability whether they are infinitely foldable or not or both. At the end I gave a conjecture.

Let's start with definitions

Definitions:

1. **Folding:** By Folding of a polygon or circle mean folding around any line of symmetry by putting either side of line on top of the other side, in such a way that area of the daughter polygon will become half of the area of parent polygon. The daughter polygon may or not be symmetrical. It is a sort of transformation.
2. **Breaking Symmetry:** If by applying **Folding** the daughter polygon have no more symmetrical line then it is called ***breaking symmetry***.
3. **Line of Folding:** The line of symmetry around which folding is done is known as line of folding.
4. Infinite and finite foldable: If there is always a line of symmetry present as area of the polygon approaches to zero then such a polygon is known as infinite foldable otherwise it will be called finite foldable.

Classification of Folding

1. Infinite foldable objects
2. Finite foldable objects
3. Both finite and finite foldable depends on line of folding.

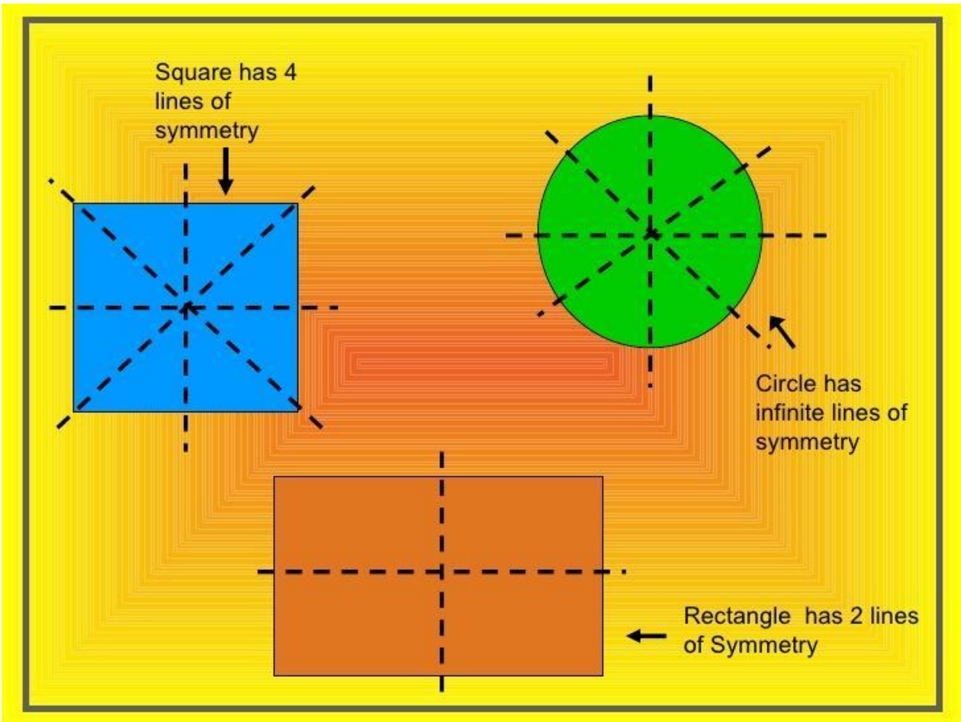
Axioms

1. Portion of the shape which will come on top of the other half will be thought of as disappearing in the other half without increasing the width of the daughter polygon.

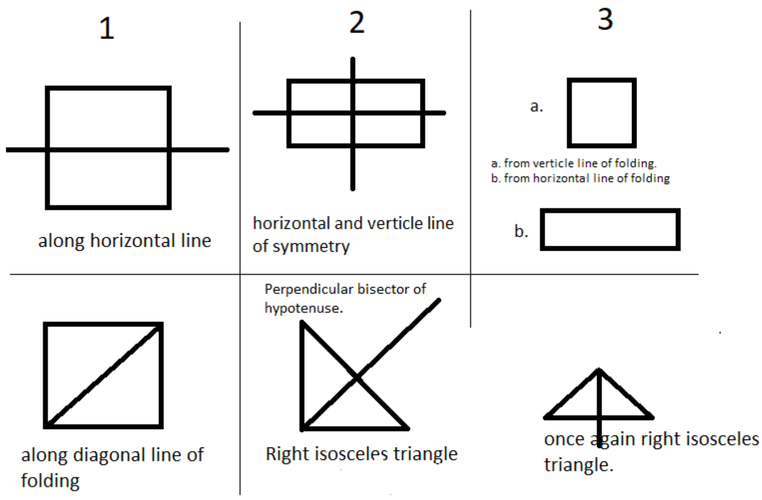
Description

1. Infinite foldable

Consider a square, rectangle (whose length is half of its width) and a circle. Square has four lines of symmetries, two horizontal and vertical perpendicular bisectors of sides and two diagonal lines of symmetries. Rectangle has two lines of symmetries along diagonal. While circle has infinitely many lines of symmetries passing through the center.



All of these shapes are examples of infinite foldable about any line of folding.



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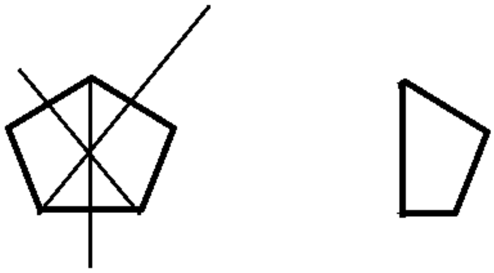
2. **Finite foldable**

All polygons apart from discussed above are finitely foldable. For example take a pentagon.



1. A regular pentagon

2. Simple 4 sided gon.

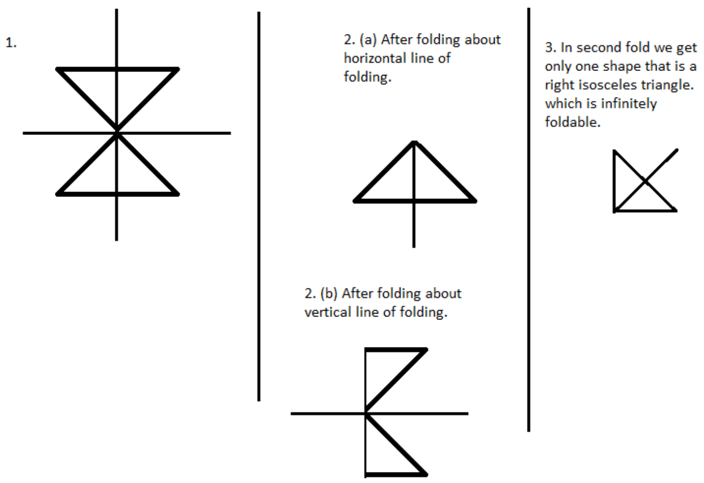
Since the number of sides of pentagon decreases from 5 to 4 after applying a fold. Thus it will not further foldable as it has no line of symmetry.

3. Both Finite and infinite foldable

Now there are some shapes of triangles which are finite or infinite foldable dependent on angle. For example an isosceles triangle having a right angle between equal sides is infinitely foldable while an isosceles triangle having no right angle is finitely foldable.

Complex polygons

These are the polygons obtained by joining regular polygons head to head or side by side. Consider this complex polygon obtained by applying the permutation (1)(2)(3 4) on a square. This will twist the square in such a way that a square will get the form of two right isosceles triangles joined together at vertex of right angle as shown in this figure. It is finitely foldable if we get this complex polygon by applying the given permutation on a rectangle. As rectangle will form non right-angled isosceles triangles which are finitely foldable. Thus, this type of complex polygon belongs to 3^{rd} category i.e. both finite and infinite foldable.



Theorem

A polygon is infinite foldable if and only if its daughter polygon retains the parent polygon or reduces to a right isosceles triangle.

Proof

Let ***P*** be a regular polygon having ***n*** sides. And ***α*** be the transformation which folds the polygon along any line of symmetry.

Case 1: If parent polygon retains its shape then it is infinite foldable.

We can write:

$\alpha (P)=P; \qquad (1)$

Apply second time,

$\alpha^2 (P)=P;$

Similarly, we always get P we can deduce that it is infinite foldable.

Case 2: If parent polygon reduces to a right isosceles triangle.

This can be proved in the same way by replacing P by R on right side of equation 1. where R represents a right isosceles triangle.

$\alpha (P)=R;$

$\alpha^2 (P)= \alpha (R)=R;$

Conjecture

No other polygon apart from the given above is infinitely foldable.