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GENERALIZED FUZZY ψ^*-CLOSED SETS IN FUZZY BITOPOLOGICAL SPACES

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ABSTRACT

In this paper, we introduce and study a new class of fuzzy sets in a fuzzy bitopological space(X, τ_1, τ_2), namely, ij-fuzzy ψ^*-closed sets, which settled properly in between the class of ji-fuzzy α-closed sets and the class of ij-fuzzy $g\alpha$-closed sets. We also introduce and study new classes of spaces, namely, ij- $FT_{1/5}$ spaces, ij- FT_e spaces, ij- $F\alpha T_e$ spaces, ij- FT_l spaces and ij-$F\alpha T_l$ spaces. As applications of ij-fuzzy ψ^*-closed sets, we introduce and study four new classes of spaces, namely, ij- $FT_{1/5}^{\psi^*}$ spaces, ij-${\psi^*}_{FT_{1/5}}$ spaces (both classes contain the class of ij- $FT_{1/5}$ spaces), ij- $F\alpha T_k$ spaces and ij- FT_k spaces. The class of ij- FT_k spaces is properly placed in between the class of ij- FT_e spaces and the class of ij- FT_l spaces. It is shown that dual of the class of ij- $FT_{1/5}^{\psi^*}$ spaces to the class of ij- $F\alpha T_e$ spaces is the class of ij- $F\alpha T_k$ spaces and the dual of the class of ij- ${\psi^*}_{FT_{1/5}}$ spaces to the class of ij- $FT_{1/5}$ spaces is the class of ij- $FT_{1/5}^{\psi^*}$ spaces and also that the dual of the class ij- FT_l spaces to the class of ij- FT_k spaces is the class of ij- $F\alpha T_k$ spaces. Further we introduce and study ij-fuzzy ψ^*continuous functions and ij-fuzzy ψ^* irresolute functions.

KEYWORDS: ij-fuzzy ψ^*-closed sets, ij-fuzzy ψ^*-continuous functions, ij- $FT_{1/5}$ spaces, ij- $FT_{1/5}^{\psi^*}$ spaces, ij-${\psi^*}_{FT_{1/5}}$ spaces.

INTRODUCTION

Recently the fuzzy topological structure τ on a set X has a lot of applications in many real life applications. The abstractness of a set X enlarges the range of its applications. For example, a special type of this fuzzy topological structure is the basic topological structure for fuzzy rough set theory and moreover, τ and its generalizations are applied in biochemical studies [1-3].

The work presented in this paper will open the way for using two viewpoints in these applications. That is, to apply two topologies at the same time. The concepts of fg-closed sets, fgs-closed sets, fsg-closed sets, $fg\alpha$-closed sets, $f\alpha g$-closed sets, fgp-closed sets, $fgsp$-closed sets and $fspg$- closed sets have been introduced in fuzzy topological spaces ([4-10]).

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Ismail Ibedou [11] introduced the concepts of ij- $FGC(X)$, ij- $FGSC(X)$, ij- $FSGC(X)$, ij- $FG\alpha C(X)$, ij- $F\alpha GC(X)$, ij- $FGPC(X)$, ij- $FGSPC(X)$ and ij- $FSPGC(X)$ subset of (X, τ_1, τ_2). Abd Allah and Nawar [12] introduced the concept of fuzzy ψ^*-open sets and studied the properties of $FT_{1/5}, FT_e, F\alpha T_e, FT_l, F\alpha T_l$. In this paper, we introduce a new class of fuzzy sets in a fuzzy bitopological space (X, τ_1, τ_2), namely, ij-fuzzy ψ^* closed sets, which settled properly in between the class of ji-fuzzy α-closed sets and the class of ij-fuzzy $g\alpha$-closed sets. And we extend the properties to a fuzzy bitopological space (X, τ_1, τ_2).

Also we use the family of ij-fuzzy ψ^*-closed sets to introduce some types of properties in (X, τ_1, τ_2), and we study the relation between these properties. The concepts of fuzzy pre-continuous, fuzzy semi-continuous, fuzzy α-continuous, fuzzy sp-continuous, fuzzy g-continuous, fuzzy αg-continuous, fuzzy $g\alpha$-continuous, fuzzy gs-continuous, fuzzy sg-continuous, fuzzy gsp-continuous, fuzzy spg-continuous, fuzzy gp-continous, fuzzy gc-irresolute, fuzzy gs-irresolute, fuzzy αg-irresolute and fuzzy $g\alpha$-irresolute functions have been introduced in fuzzy topological spaces ([7,10, 13-28]). Ismail Ibedou [11] introduced the concepts of (ij-fuzzy pre-continuous, ij-fuzzy semi-continuous, ij-fuzzy α-continuous, ij-fuzzy sp-continuous, ij-fuzzy g-continuous, ij- fuzzy αg-continuous, ij-fuzzy $g\alpha$-continuous, ij-fuzzy gs-continuous, ij-fuzzy sg-continuous, ij-fuzzy gsp-continuous, ij-fuzzy spg-continuous, ij-fuzzy gp-continuous, ij-fuzzy gc-irresolute, ij-fuzzy gs-irresolute, ij-fuzzy αg-irresolute, ij-fuzzy $g\alpha$-irresolute) functions in fuzzy bitopological spaces. In this paper, we introduce a new functions in a fuzzy bitopological space (X, τ_1, τ_2), namely, ij-fuzzy ψ^*-continuous functions and ij-fuzzy ψ^*-irresolute functions.

PRELIMINARIES

DEFINITION 2.1 [23]

A fuzzy subset A of a fuzzy bitopological space (X, τ_1, τ_2) is called:

(1) ij-fuzzy preopen if $A \leq \tau_i\text{-}int\left(\tau_j - cl(A)\right)$ and ij-fuzzy preclosed if $\tau_i\text{-}cl\left(\tau_j - int(A)\right) \leq A$.
(2) ij-fuzzy semi-open if $A \leq \tau_j\text{-}cl\left(\tau_i - int(A)\right)$ and ij-fuzzy semi-closed if $\tau_j\text{-}int\left(\tau_i - cl(A)\right) \leq A$.
(3) ij-fuzzy α-open if $A \leq \tau_i\text{-}int\left(\tau_j - cl\left(\tau_i - int(A)\right)\right)$ and ij-fuzzy α-closed if $\tau_i\text{-}cl\left(\tau_j - int\left(\tau_i - cl(A)\right)\right) \leq A$.
(4) ij-fuzzy semi-preopen if $A \leq \tau_j\text{-}cl\left(\tau_i - int\left(\tau_j - cl(A)\right)\right)$ and ij-fuzzy semi preclosed if $\tau_j\text{-}int\left(\tau_i - cl\left(\tau_j - int(A)\right)\right) \leq A$.

The class of all ij-fuzzy preopen (resp. ij-fuzzy semi-open, ij-fuzzy α-open and ij- fuzzy semi-preopen) sets in a fuzzy bitopological space (X, τ_1, τ_2) is denoted by ij- $FPO(X)$ (resp. ij- $FSO(X)$, ij-$F\alpha O(X)$ and ij- $FSPO(X)$). The class of all ij-fuzzy preclosed (resp. ij- fuzzy semi-closed, ij-fuzzy α-closed and ij-fuzzy semi-preclosed) sets in a fuzzy bitopological space (X, τ_1, τ_2) is denoted by ij- $FPC(X)$ (resp. ij-$FSC(X)$, ij-$F\alpha C(X)$ and ij- $FSPC(X)$).

DEFINITION 2.2 [23]

For a fuzzy subset A of a fuzzy bitopological space (X, τ_1, τ_2), the ij-fuzzy pre-closure (resp. ij-fuzzy semi-closure, ij-fuzzy α-closure and ij-fuzzy semi-pre-closure) of A are denoted and defined as follow:

(1) $ij - fpcl(A) = \wedge\{F < X : F \in ij - FPC(X), F \geq A\}$.

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(2) $ij - fscl(A) = \wedge \{F < X : F \in ij - FSC(X), F \geq A\}$.
(3) $ij - f\alpha cl(A) = \wedge \{F < X : F \in ij - F\alpha C(X), F \geq A\}$.
(4) $ij - fspcl(A) = \wedge \{F < X : F \in ij - FSPC(X), F \geq A\}$.

Dually, the ij-fuzzy preinterior (resp. ij-fuzzy semi-interior, ij-fuzzy α-interior and ij- fuzzy semi-preinterior) of A, denoted by $ij\text{-}fpint(A)$ (resp. $ij\text{-} fsint(A), ij\text{-}f\alpha int(A)$ and $ij\text{-}fspint(A)$) is the union of all ij-fuzzy preopen (resp. ij-fuzzy semi-open, ij-fuzzy α-open and ij-fuzzy semi-preopen) fuzzy subsets of X contained in A.

DEFINITION 2.3 [11]

A fuzzy subset A of a fuzzy bitopological space (X, τ_1, τ_2) is called:

(1) ij- fuzzy g-closed (denoted by $ij\text{-} FGC(X)$) if, $A \leq U, U \in \tau_i \Rightarrow j\text{-}fcl(A) \leq U$.
(2) ij-fuzzy gs-closed (denoted by $ij\text{-}FGSC(X)$) if, $A \leq U$, $U \in \tau_i \Rightarrow ji\text{-}fscl(A) \leq U$.
(3) ij-fuzzy sg-closed (denoted by $ij\text{-}FSGC(X)$) if, $A \leq U$, $U \in ij\text{-}FSO(X) \Rightarrow ji\text{-}fscl(A) \leq U$.
(4) ij-fuzzy $g\alpha$-closed (denoted by $ij\text{-}FG\alpha C(X)$) if, $A \leq U$, $U \in ij\text{-}F\alpha O(X) \Rightarrow ji\text{-}f\alpha cl(A) \leq U$.
(5) ij-fuzzy αg-closed (denoted by $ij\text{-}F\alpha GC(X)$) if, $A \leq U$, $U \in \tau_i \Rightarrow ji\text{-}F\alpha cl(A) \leq U$.
(6) ij-fuzzy gp-closed (denoted by $ij\text{-}FGPC(X)$) if, $A \leq U$, $U \in \tau_i \Rightarrow ji\text{-}fpcl(A) \leq U$.
(7) ij-fuzzy gsp-closed (denoted by $ij\text{-}FGSPC(X)$) if, $A \leq U, U \in \tau_i \Rightarrow ji\text{-}fspcl(A) \leq U$.
(8) ij-fuzzy spg-closed (denoted by $ij\text{-}FSPGC(X)$)) if, $A \leq U$, $U \in ji\text{-}FSPO(X) \Rightarrow ji\text{-}fspcl(A) \leq U$.

The fuzzy complement of an $ij\text{-}FGC(X)$ (resp. $ij\text{-}FGSC(X)$, $ij\text{-}FSGC(X), ij\text{-}FG\alpha C$, $ij\text{-}F\alpha GC(X)$, $ij\text{-}FGPC(X)$, $ij\text{-}FGSPC(X)$ and $ij\text{-}FSPGC(X)$) fuzzy subset of (X, τ_1, τ_2) is called an $ij\text{-} FGO(X)$ (resp. $ij\text{-}FGSO(X)$, $ij\text{-}FSGO(X)$, $ij\text{-}FG\alpha O(X)$, $ij\text{-}F\alpha GO(X)$, $ij\text{-}FGPO(X)$, $ij\text{-}FGSPO(X)$ and $ij\text{-} FSPGO(X)$) fuzzy subset of (X, τ_1, τ_2).

DEFINITION 2.4[11]

A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called:

(1) ij-fuzzy pre-continuous if $\forall V \in i\text{-}FC(Y), f^{-1}(V) \in ij\text{-}FPC(X)$.
(2) ij-fuzzy semi-continuous if $\forall V \in i\text{-}FC(Y), f^{-1}(V) \in ij\text{-}FSC(X)$.
(3) ij-fuzzy α-continuous if $\forall V \in i\text{-}FC(Y), f^{-1}(V) \in ij\text{-}F\alpha C(X)$.
(4) ij- fuzzy sp-continuous if $\forall V \in i\text{-}FC(Y), f^{-1}(V) \in ij\text{-}FSPC(X)$.
(5) ij-fuzzy g-continuous if $\forall V \in j\text{-}FC(Y), f^{-1}(V) \in ij\text{-}FGC(X)$.
(6) ij-fuzzy αg-continuous if $\forall V \in j\text{-}FC(Y), f^{-1}(V) \in ij\text{-}F\alpha GC(X)$.
(7) ij-fuzzy $g\alpha$-continuou if $\forall V \in j\text{-}FC(Y), f^{-1}(V) \in ij\text{-}FG\alpha C(X)$.
(8) ij-fuzzy gs-continuous if $\forall V \in j\text{-}FC(Y), f^{-1}(V) \in ij\text{-}FGSC(X)$.
(9) ij-fuzzy sg-continuous if $\forall V \in j\text{-}FC(Y), f^{-1}(V) \in ij\text{-}FSGC(X)$.
(10) ij-fuzzy gsp-continuous if $\forall V \in j\text{-}FC(Y), f^{-1}(V) \in ij\text{-}FGSPC(X)$.
(11) ij-fuzzy spg-continuous if $\forall V \in j\text{-}FC(Y), f^{-1}(V) \in ij\text{-}FSPGC(X)$.
(12) ij-fuzzy gp-continuous if $\forall V \in j\text{-}FC(Y), f^{-1}(V) \in ij\text{-}FGPC(X)$.
(13) i-continuous if $\forall V \in i\text{-}FC(Y), f^{-1}(V) \in i\text{-}FC(X)$.
(14) ij-fuzzy gc-irresolute if $\forall V \in ij\text{-}FGC(Y), f^{-1}(V) \in ij\text{-}FGC(X)$.
(15) ij-fuzzy gs-irresolute if $\forall V \in ij\text{-}FGSC(Y), f^{-1}(V) \in ij\text{-}FGSC(X)$.
(16) ij-fuzzy αg-irresolute if $\forall V \in ij\text{-}F\alpha GC(Y), f^{-1}(V) \in ij\text{-}F\alpha GC(X)$.

(17) ij-fuzzy $g\alpha$-irresolute if $\forall V \in ij\text{-}FG\alpha C(Y), f^{-1}(V) \in ij\text{-}FG\alpha C(X)$.

DEFINITION 2.5 [12]

A fuzzy subset A of (X,τ) is called fuzzy ψ^*-closed if $A \le U, U \in FG\alpha O(X) \Rightarrow f\alpha cl(A) \le U$. The fuzzy complement of fuzzy ψ^*-closed set is said to be fuzzy ψ^*-open.

DEFINITION 2.6 [12]

A fuzzy topological space (X,τ) is called:

(1) $FT_{1/5}$ space if $FG\alpha C(X) = F\alpha C(X)$.
(2) $FT_{1/5}^{\psi^*}$ space if $F\psi^* C(X) = F\alpha C(X)$.
(3) ${\psi^*}_{FT_{1/5}}$ space if $FG\alpha C(X) = F\psi^* C(X)$.
(4) FT_e space if $FGSC(X) = F\alpha C(X)$.
(5) $F\alpha T_e$ space if $F\alpha GC(X) = F\alpha C(X)$.
(6) FT_k space if $FGSC(X) = F\psi^* C(X)$.
(7) $F\alpha T_k$ space if $F\alpha GC(X) = F\psi^* C(X)$.
(8) FT_l space if $FGSC(X) = FG\alpha C(X)$.
(9) $F\alpha T_l$ space if $F\alpha GC(X) = FG\alpha C(X)$.

DEFINITION 2.7 [12]

A function $f : (X,\tau) \to (Y,\sigma)$ is called:

(1) Fuzzy ψ^*-continuous if $\forall V \in FC(Y), f^{-1}(V) \in F\psi^* C(X)$.
(2) Fuzzy ψ^*-irresolute if $\forall V \in F\psi^* C(Y), f^{-1}(V) \in F\psi^* C(X)$.
(3) Fuzzy pre-ψ^*-closed if $A \in F\psi^* C(X), f(A) \in F\psi^* C(Y)$.

3. BASIC PROPERTIES IF ij-FUZZY ψ^*-CLOSED SETS

We introduce the following definition.

DEFINITION 3.1

A fuzzy subset A of a fuzzy bitopological space (X,τ_1,τ_2) is called ij-fuzzy ψ^*-closed set if, $A \le U, U \in ji\text{-}FG\alpha O(X) \Rightarrow ji\text{-}f\alpha cl(A) \le U$.

The class of ij-fuzzy ψ^*-closed subsets of (X,τ_1,τ_2) is denoted by $ij\text{-}F\psi^* C(X)$.

The following diagram shows the relationships of ij-fuzzy ψ^*-closed sets with some other fuzzy sets discussed in this section (Diagram 1).

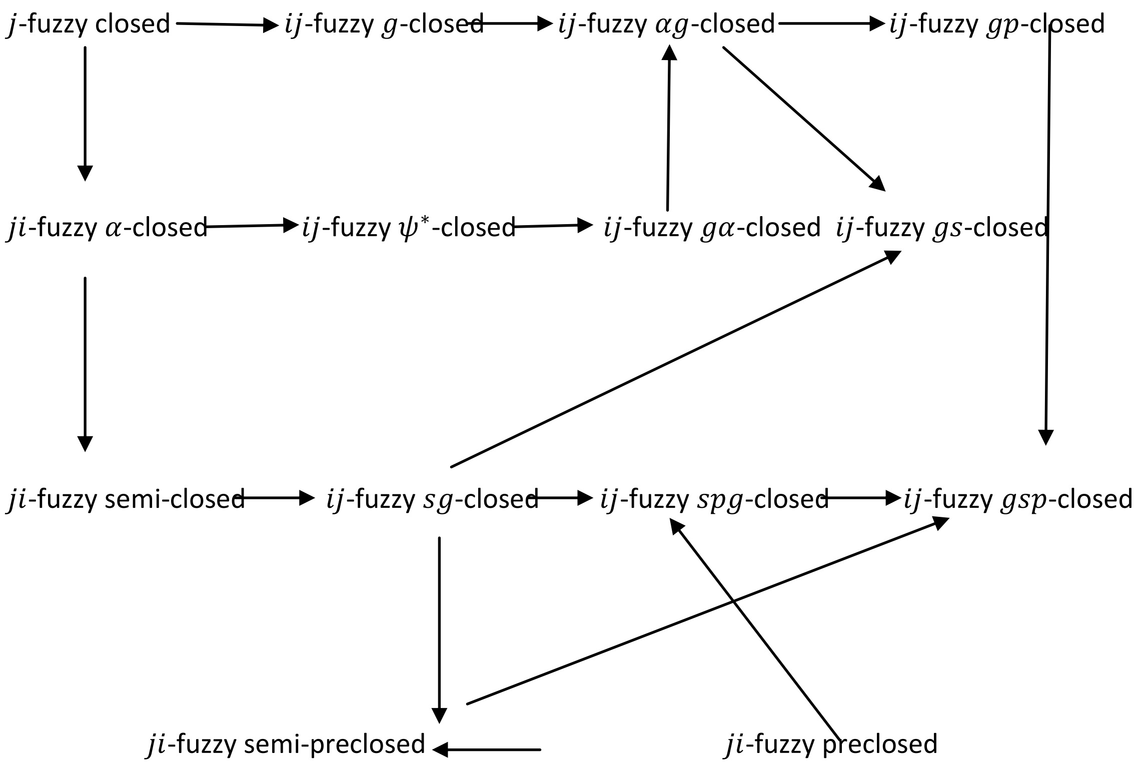

Diagram 1

EXAMPLE 3.1

Let $X = \{a, b, c\}$

$Y = \{p, q\}$

$\tau_1 = \{0, 1, \alpha_1, \alpha_2, \alpha_3\}$

$\tau_2 = \{0, 1, \beta\}$

$\alpha_1 = \frac{0.6}{a} + \frac{0}{b} + \frac{0}{c}$

$\alpha_2 = \frac{0}{a} + \frac{0.6}{b} + + \frac{0}{c}$

$\alpha_3 = \frac{0.6}{a} + \frac{0.6}{b} + \frac{0}{c}$

And $\beta = \frac{0.6}{p} + \frac{0}{q}$

β: $(X, \tau_1) \rightarrow (Y, \tau_2)$

As follows: $f(a) = p$, $f(b) = f(c) = q$.

Then $\beta \in 12\text{-}Fgsp$-closed but $\alpha_1 \wedge \alpha_3 = \alpha_3 \notin 12\text{-}Fgsp$-closed.

Where none of these implications is reversible as shown by the following example.

EXAMPLE 3.1.1

Let $I = [0,1]$ and $\sigma_1, \sigma_2, \sigma_3$ be fuzzy sets of I defined as

$$\sigma_1(x) = \begin{cases} 0 \text{ if } 0 \le x \le \frac{1}{2} \\ x - 2 \text{ if } \frac{1}{2} \le x \le 1 \end{cases}$$

$$\sigma_2(x) = \sigma_3(x) = \begin{cases} 1 \text{ if } 0 \le x \le \frac{1}{4} \\ 4x \text{ if } \frac{1}{4} \le x \le \frac{1}{2} \\ 0 \text{ if } \frac{1}{2} \le x \le 1 \end{cases}$$

Clearly $\tau_1 = \{0,1, \sigma_1, \sigma_2, \sigma_1 \vee \sigma_2\}$ and $\tau_2 = \{0,1, \sigma_3\}$ are fuzzy topologies on I.

Let $f: (I, \tau_1) \rightarrow (I, \tau_2)$ be defined by $f(x) = x$ for $x \in I$

(arrows 1,5) $\sigma_3 \in 12\text{-}Fg$-closed $\wedge$ $12\text{-}Fg\alpha$-closed but $\sigma_3 \notin 2$-Fuzzy closed.

(arrows 2.6) $\sigma_3 \in 12\text{-}F\alpha g$-closed $\wedge$ $12\text{-}Fsg$-closed but $\sigma_3 \notin 21\text{-}F\alpha$-closed, since there exist $\sigma_1 \vee \sigma_2 \in \tau_1$ containing σ_1 such that $2\text{-}cl\{\sigma_1\} = \sigma_3 \notin \tau_1$.

(arrows 3,7) $\sigma_3 \in 12\text{-}Fgp$-closed $\wedge$ $12\text{-}Fspg$-closed but $\sigma_3 \notin 21$-Fuzzy semi closed, since there exist $\sigma_3 \in \tau_2$ containing σ_3 such that $21\text{-}F\alpha$-closed $\sigma_3 = X \nleq \sigma_3$.

(arrow 4) $\sigma_3 \in 12\text{-}F\psi$-closed but $\sigma_3 \notin 21$-Fuzzy closed.

(arrow 8) $\sigma_3 \in 12\text{-}Fgsp$-closed but $\sigma_3 \notin 21$-Fuzzy semi closed, since there exist $\sigma_1 \vee \sigma_2 \in 21$-Fuzzy semi generalized closed containing σ_1 such that $1\text{-}cl\{\sigma_1 \vee \sigma_2\} = (\sigma_1 \vee \sigma_2)^c \nleq \sigma_1$.

(arrow 9) $\sigma_3 \in 21$-Fuzzy α-closed but $\sigma_3 \notin 2$-Fuzzy closed.

(arrow 10) $\sigma_3 \in 12\text{-}F\alpha g$-closed but $\sigma_3 \notin 2$-Fuzzy closed, since there exist $\sigma_3 \in \tau_2$ containing σ_3 such that 2-Fuzzy closed, $\sigma_3 = X \nleq \sigma_3$.

(arrow 11) $\sigma_3 \in 12\text{-}Fgs$-closed but $\sigma_3 \notin 21$-Fuzzy closed, since there exist $\sigma_3 \in \tau_2$ containing σ_3 such that 12-Fuzzy ψ-closed, $\sigma_3 = X \nleq \sigma_3$.

(arrow 12) $\sigma_3 \in 21$-Fuzzy semi closed but $\sigma_3 \notin 21$-Fuzzy α-closed.

(arrow 13) $\sigma_3 \in 12\text{-}Fgsp$-closed but $\sigma_3 \notin 21$-Fuzzy semi closed, since there exist $\sigma_1 \vee \sigma_2 \in \tau_1$ containing σ_1 such that $2\text{-}cl\{\sigma_1\} = \sigma_3 \notin \tau_1$.

(arrow 14) $\sigma_3 \in 21$-Fuzzy semi-pre-closed but 21-Fuzzy semi closed.

(arrow 15) $\sigma_3 \in 12$-Fuzzy gsp-closed but 21-Fuzzy semi closed.

(arrow 16) $\sigma_3 \in 21$-Fuzzy pre-closed but $\sigma_3 \notin 12$-Fuzzy gp-closed.

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(arrow 17) $\sigma_3 \in$ *12-Fgsp*-closed but $\sigma_3 \notin$ *12-*Fuzzy closed, since there exist $\sigma_1 \vee \sigma_2 \in$ *21-Fα*-open containing σ_1 such that *1-cl* $\{\sigma_1 \vee \sigma_2\} = (\sigma_1 \vee \sigma_2)^c \nleq \sigma_1$.

(arrow 18) $\sigma_3 \in$ *12-Fgs*-closed but $\sigma_3 \notin$ *21-*Fuzzy *α*-closed.

THEOREM 3.1

Every *ij*-fuzzy *α*-closed set is an *ij*-fuzzy *ψ̂*-closed set.

The following example supports that an *ij*-fuzzy *ψ̂*-closed set need not be a *ij*-fuzzy *α*-closed set in general.

EXAMPLE 3.1.2

Let $X = \{a, b, c, d\}$, $\tau_1 = \{X, \phi, \{a\}, \{a, d\}\}$ and $\tau_2 = \{X, \phi, \{a, b\}, \{c, d\}\}$. Then we have $A = \{b, c\} \in$ *ij-Fψ̂ C(X)* but $A \notin$ *ji-FαC(X)*.

Therefore the class of *ij*-fuzzy *ψ̂*-closed sets is properly contains the class of *ji*-fuzzy *α*-closed sets. Next we show that the class of *ij*-fuzzy *ψ̂*-closed sets is properly contained in the class of *ij*-fuzzy *gα*-closed set.

THEOREM 3.2

Every *ij*-fuzzy *ψ̂*-closed set is an *ij*-fuzzy *gα*-closed set.

The following example supports that the converse of the above theorem is not true in general.

EXAMPLE 3.2

Let X, τ_1 and τ_2 are as in Example 3.1. Then the fuzzy subset $B = \{b\} \in$ *ij-FGαC(X)* but $B \notin$ *ij-Fψ̂ C(X)*.

REMARK 3.1

The fuzzy intersection of two sets in *ij*-fuzzy *ψ̂*-closed set is not in general a set in *ij*-fuzzy *ψ̂*-closed set, as shown by the following example.

EXAMPLE 3.3

Let X, τ_1 and τ_2 be as in the Example 3.1. Then we have $\{a, b\}$ and $\{b, c\} \in$ *ij-Fψ̂ C(X)* but $\{a, b\} \wedge \{b, c\} = \{b\} \notin$ *ij-Fψ̂ C(X)*.

THEOREM 3.3

For any fuzzy bitopological space (X, τ_1, τ_2).

(1) *ij-Fψ̂ C(X)* $\wedge$ *ji-FGαO(X)* $\leq$ *ji-FαC(X)*.
(2) If $A \in$ *ij-Fψ̂ C(X)* and $A \leq B \leq$ *ji-fαcl* (A), then $B \in$ *ij-Fψ̂ C(X)*.

PROOF.

(1) Let $A \in$ *ij-Fψ̂ C(X)* $\wedge$ *ji-FGαO(X)*. Then we have *ji-fαcl* $(A) \leq A$. Consequently, $A \in$ *ji-FαC(X)*.
(2) Let $U \in$ *ji-FGαO(X)* such that $B \leq U$. Since $A \leq B$ and $A \in$ *ij-Fψ̂ C(X)*, then *ji-fαcl* $(A) \leq U$. Since $B \leq$ *ji-fαcl* (A), then we have *ji-fαcl* $(B) \leq$ *ji-fαcl* $(A) \leq U$. Therefore, $B \in$ *ij-Fψ̂ C(X)*.

THEOREM 3.4

Let (X, τ_1, τ_2) be a fuzzy bitopological space, $A \in$ *ij-FGαC(X)*. Then $A \in$ *ij-Fψ̂ C(X)* if *ij-FαO(X)* $=$ *ji*-$F G\alpha O(X)$.

PROOF.

Let $A \in ij\text{-}FG\alpha C(X)$ i.e. $A \leq U$ and $U \in ij\text{-}F\alpha O(X)$, then $ji\text{-}f\alpha cl(A) \leq U$. Since $ij\text{-}F\alpha O(X) = ji\text{-}FG\alpha O(X)$. Consequently, $A \leq U$ and $U \in ji\text{-}FG\alpha O(X)$, then $ji\text{-}f\alpha cl(A) \leq U$ i.e. $A \in ij\text{-}F\psi^* C(X)$.

THEOREM 3.5

Let (X_1, τ_1, τ_2) and $(X_1, \tau_1^*, \tau_2^*)$ be two fuzzy bitopological spaces. Then the following statement is true. If $A \in ij\text{-}F\psi^* O(X_1)$ and $B \in ij\text{-}F\psi^* O(X_2)$, then $A \times B \in ij\text{-}F\psi^* O(X_1 \times X_2)$.

PROOF.

Let $A \in ij\text{-}F\psi^*O(X_1)$ and $B \in ij\text{-}F\psi^*O(X_2)$ and $W = A \times B \leq X_1 \times X_2$. Let $F = F_1 \times F_2 \leq W, F \in ji\text{-}FG\alpha C(X_1 \times X_2)$. Then there are $F_1 \in ji\text{-}FG\alpha C(X_1)$, $F_2 \in ji\text{-}FG\alpha C(X_2), F_1 \leq A$, $F_2 \leq B$ and so, $F_1 \leq \tau_{ji}\text{-}f\alpha int(A)$ and $F_2 \leq \tau_{ji}^*\text{-}f\alpha int(B)$. Hence $F_1 \times F_2 \leq A \times B$ and $F_1 \times F_2 \leq \tau_{ji}\text{-}f\alpha int(A) \times \tau_{ji}^*\text{-}f\alpha int(B) = \tau_{ji} \times \tau_{ji}^*\text{-}f\alpha int(A \times B)$.

Therefore $A \times B \in ij\text{-}\ F\psi^*O(X_1 \times X_2)$.

THEOREM 3.6

A fuzzy subset A of X is $ij\text{-}F\psi^*O(X)$ if and only if F is a fuzzy subset of $ij\text{-}f\alpha int(A)$ whenever $F \leq A$ and $F \in ji\text{-}FG\alpha C(X)$.

THEOREM 3.7

For each $x \in X$, either $\{x\}$ is $ji\text{-}FG\alpha C(X)$ or $\{x\}$ is $ij\text{-}F\psi^*O(X)$.

THEOREM 3.8

A fuzzy subset A of X is $ij\text{-}F\psi^*C(X)$ if and only if $ji\text{-}F\alpha C(A) \wedge F = \emptyset$, whenever $A \wedge F = \emptyset$, where F is $ji\text{-}FG\alpha C(X)$.

APPLICATIONS OF ij- FUZZY ψ^*-CLOSED SETS

As applications of ij-fuzzy ψ^*-closed sets, four new classes of spaces, namely, $ij\text{-}FT_{1/5}^{\psi^*}$ spaces, $ij\text{-}\psi^*{}_{FT_{1/5}}$spaces, $ij\text{-}FT_k$ spaces, and $ij\text{-}F\alpha T_k$ spaces are introduced.

We introduce the following definitions.

DEFINITION 4.1

A fuzzy bitopological space (X, τ_1, τ_2) is called an $ij\text{-}FT_{1/5}$ space if $ij\text{-}FG\alpha C(X) = ji\text{-}F\alpha C(X)$.

EXAMPLE 4.1

Let $X = \{a, b, c, d\}$

$\tau_1 = \{X, \phi, \{a, b\}\}$

$\tau_2 = \{X, \phi, \{b\}, \{c\}, \{b, c\}, \{a, c\}\}$

Then the sets in $\{X, \phi, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ are called $ij\text{-}FT_{1/5}$ open and the sets in $\{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}\}$ are called $ij\text{-}FT_{1/5}$ closed.

Then $(1,5)\text{-}\ ij\text{-}F\psi^*C(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}\}$.

Clearly the sets $\{b\}$ and $\{c\}$ are $(1,5)$-Fuzzy ψ^*-closed but their union $\{b, c\}$ is not $(1,5)$-Fuzzy ψ^*-closed set in X.

DEFINITION 4.2

A fuzzy bitopological space (X, τ_1, τ_2) is called an $ij\text{-}FT_{1/5}^{\psi^*}$ space if $ij\text{-}F\psi^*C(X) = ji\text{-}F\alpha C(X)$.

We prove that the class of $ij\text{-}FT_{1/5}^{\psi^*}$ spaces properly contains the class of $ij\text{-}FT_{1/5}$ spaces.

THEOREM 4.1

Every $ij\text{-}FT_{1/5}$ space is an $ij\text{-}FT_{1/5}^{\psi^*}$ space.

PROOF.

Follows from the fact that every ij-fuzzy ψ^*-closed set is an ij-fuzzy $g\alpha$-closed set.

The converse of the above theorem is not true as it can be seen from the following example.

EXAMPLE 4.2

Let $X = \{a, b, c\}, \tau_1 = \{X, \phi, \{a\}\}$ and $\tau_2 = \{X, \phi, \{b\}\}$. Then (X, τ_1, τ_2) is an ij-

$FT_{1/5}^{\psi^*}$ space but not an $ij\text{-}FT_{1/5}$ space since $\{b, c\} \in ij\text{-}FG\alpha C(X)$ but $\{b, c\} \notin ji\text{-}F\alpha C(X)$.

DEFINITION 4.3

A fuzzy bitopological space(X, τ_1, τ_2) is called an $ij\text{-}\psi^*{}_{FT_{1/5}}$ space if $ij\text{-}FG\alpha C(X) = ij\text{-}F\psi^*C(X)$.

THEOREM 4.2

Every $ij\text{-}FT_{1/5}$ space is an $ij\text{-}\psi^*{}_{FT_{1/5}}$ space.

PROOF.

Let (X,τ_1,τ_2) be an $ij\text{-}FT_{1/5}$ space. Let $A \in ij\text{-}FG\alpha C(X)$. Since (X,τ_1,τ_2) is an $ij\text{-}FT_{1/5}$ space, then $A \in ji\text{-}F\alpha C(X)$. Hence, by using Theorem 3.1, we have $A \in ij\text{-}F\psi^* C(X)$. Therefore (X,τ_1,τ_2) is an $ij\text{-}\psi^*{}_{FT_{1/5}}$ space.

The converse of the above theorem is not true as we see in the following example.

EXAMPLE 4.3

Let $X = \{a,b,c\}, \tau_1 = \{X,\phi,\{a\}\}$ and $\tau_2 = \{X,\phi,\{a\},\{b,c\}\}$. Then (X,τ_1,τ_2) is an $ij\text{-}\psi^*{}_{FT_{1/5}}$ space but not an $ij\text{-}FT_{1/5}$ space since $\{a,b\} \in ij\text{-}FG\alpha C(X)$ but $\{a,b\} \notin ji\text{-}F\alpha C(X)$.

We show that $ij\text{-}FT_{1/5}^{\psi^*}$ ness is independent from $ij\text{-}\psi^*{}_{FT_{1/5}}$ ness.

REMARK 4.1

$ij\text{-}FT_{1/5}^{\psi^*}$ ness and $ij\text{-}\psi^*{}_{FT_{1/5}}$ ness are independent as it can be seen from the next two examples.

EXAMPLE 4.4

Let X,τ_1 and τ_2 be as in the Example 4.1. Then (X,τ_1,τ_2) is an $ij\text{-}FT_{1/5}^{\psi^*}$ space but not an $ij\text{-}\psi^*{}_{FT_{1/5}}$ space since $\{b,c\} \in ij\text{-}FG\alpha C(X)$ but $\{b,c\} \notin ij\text{-}F\psi^* C(X)$.

EXAMPLE 4.5

Let X,τ_1 and τ_2 be as in the Example 4.2. Then (X,τ_1,τ_2) is an $ij\text{-}\psi^*{}_{FT_{1/5}}$ space but not an $ij\text{-}FT_{1/5}^{\psi^*}$ space since $\{a,c\} \in ij\text{-}F\psi^* C(X)$ but $\{a,c\} \notin ji\text{-}F\alpha C(X)$.

THEOREM 4.3

If (X,τ_1,τ_2) is an $ij\text{-}\psi^*{}_{FT_{1/5}}$ space, then for each $x \in X, \{x\}$ is either ij-fuzzy α-closed or ij-fuzzy ψ^*-open.

PROOF.

Suppose that (X,τ_1,τ_2) is an $ij\text{-}\psi^*{}_{FT_{1/5}}$ space. Let $x \in X$ and assume that $\{x\} \notin ji\text{-}F\alpha C(X)$. Then $\{x\} \notin ij\text{-}FG\alpha C(X)$ since every ji-fuzzy α-closed set is an ij-fuzzy $g\alpha$-closed set. So X-$\{x\} \notin ji\text{-}F\alpha O(X)$. Therefore X-$\{x\} \in ij\text{-}FG\alpha C(X)$ since X is the only ji-fuzzy α-open set which contains X-$\{x\}$. Since (X,τ_1,τ_2) is an $ij\text{-}\psi^*{}_{FT_{1/5}}$ space, then X-$\{x\} \in ij\text{-}F\psi^* C(X)$ or equivalently $\{x\} \notin ij\text{-}F\psi^* O(X)$.

THEOREM 4.4

A fuzzy bitopological space (X,τ_1,τ_2) is an $ij\text{-}FT_{1/5}$ space if and only if it is $ij\text{-}\psi^*{}_{FT_{1/5}}$ and $ij\text{-}FT_{1/5}^{\psi^*}$ space.

PROOF.

The necessity follows from the Theorems 4.1 and 4.2. For the sufficiency, suppose that (X,τ_1,τ_2) is both $ij\text{-}\psi^*{}_{FT_{1/5}}$ and $ij\text{-}FT_{1/5}^{\psi^*}$ space. Let $A \in ij\text{-}FG\alpha C(X)$. Since (X,τ_1,τ_2) is an $ij\text{-}\psi^*{}_{FT_{1/5}}$ space, then $A \in ij\text{-}F\psi^* C(X)$. Since (X,τ_1,τ_2) is an $ij\text{-}FT_{1/5}^{\psi^*}$ space, then $A \in ji\text{-}F\alpha C(X)$. Thus (X,τ_1,τ_2) is an $ij\text{-}FT_{1/5}$ space.

We introduce the following definitions $ij\text{-}FT_e$ spaces and $ij\text{-}F\alpha T_e$ spaces respectively and show that every $ij\text{-}FT_e(ij - F\alpha T_e)$ space is an $ij\text{-}FT_{1/5}$ space.

DEFINITION 4.4

A fuzzy bitopological space (X,τ_1,τ_2) is called an $ij\text{-}FT_e$ space if $ij\text{-}FGSC(X) = ji\text{-}F\alpha C(X)$.

DEFINITION 4.5

A fuzzy bitopological space (X, τ_1, τ_2) is called an $ij\text{-}F\alpha T_e$ space if $ij\text{-}F\alpha GC(X) = ji\text{-}F\alpha C(X)$.

THEOREM 4.5

Every $ij\text{-}FT_e$ space is an $ij\text{-}FT_{1/5}$ space.

PROOF.

Follows from the fact that every ij-fuzzy $g\alpha$-closed set is an ij-fuzzy gs-closed set.

An $ij\text{-}FT_{1/5}$ space need not be an $ij\text{-}FT_e$ space as we see the next example.

EXAMPLE 4.6

Let $X = \{a, b, c\}, \tau_1 = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ and $\tau_2 = \{X, \phi, \{a\}, \{a, b\}\}$. Then (X, τ_1, τ_2) is an $ij\text{-}FT_{1/5}$ space but not an $ij\text{-}FT_e$ space since $\{b\} \in ij\text{-}FGSC(X)$ but $\{b\} \notin ji\text{-}F\alpha C(X)$.

THEOREM 4.6

Every $ij\text{-}F\alpha T_e$ space is an $ij\text{-}FT_{1/5}$ space.

PROOF.

Follows from the fact that every ij-fuzzy $g\alpha$-closed set is an ij-fuzzy αg-closed set.

An $ij\text{-}FT_{1/5}$ space need not be an $ij\text{-}F\alpha T_e$ space as we see the next example.

EXAMPLE 4.7

Let $X = \{a, b, c\}, \tau_1 = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$ and $\tau_2 = \{X, \phi, \{a\}, \{a, c\}\}$. Then (X, τ_1, τ_2) is an $ij\text{-}FT_{1/5}$ space but not an $ij\text{-}F\alpha T_e$ space since $\{a, c\} \in ij\text{-}F\alpha GC(X)$ but $\{a, c\} \notin ji\text{-}F\alpha C(X)$.

THEOREM 4.7

Every $ij\text{-}FT_e$ space is an $ij\text{-}F\alpha T_e$ space.

PROOF.

Follows from the fact that every ij-fuzzy αg-closed set is an ij-fuzzy gs-closed set.

The converse of the above theorem is not true in general as the following example supports.

EXAMPLE 4.8

Let X, τ_1 and τ_2 be as in the Example 4.5. Then (X, τ_1, τ_2) is an $ij\text{-}F\alpha T_e$ space but not an $ij\text{-}FT_e$ space since $\{b\} \in ij\text{-}FGSC(X)$ but $\{b\} \notin ji\text{-}F\alpha C(X)$.

THEOREM 4.8

Every $ij\text{-}FT_e$ space is an $ij\text{-}FT_{1/5}^{\psi^*}$ space.

PROOF.

Follows from the fact that every ij-fuzzy ψ^*-closed set is an ij-fuzzy gs-closed set.

The converse of the above theorem is not true in general as the following example supports.

EXAMPLE 4.9

Let $X = \{a, b, c, d, e\}$, $\tau_1 = \{X, \phi, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d, e\}\}$ and $\tau_2 = \{X, \phi, \{a\}, \{a, b\}, \{a, b, e\}, \{a, c, d\}, \{a, b, c, d\}\}$. Then (X, τ_1, τ_2) is an $ij\text{-}FT_{1/5}^{\psi^*}$ space but not an $ij\text{-}FT_e$ space since $\{d\} \in ij\text{-}FGSC(X)$ but $\{d\} \notin ji\text{-}F\alpha C(X)$.

THEOREM 4.9

Every $ij\text{-}F\alpha T_e$ space is an $ij\text{-}FT_{1/5}^{\psi^*}$ space.

PROOF.

Follows from the fact that every ij-fuzzy ψ^*-closed set is an ij-fuzzy αg-closed set.

An $ij\text{-}FT_{1/5}^{\psi^*}$ space need not be an $ij\text{-}F\alpha T_e$ space as we see the next example.

EXAMPLE 4.10

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Let X, τ_1 and τ_2 be as in Example 4.8. Then (X, τ_1, τ_2) is an $ij\text{-}FT_{1/5}^{\psi^*}$ space but not an $ij\text{-}F\alpha T_e$ space $\{c\} \in ij\text{-}F\alpha GC(X)$ but $\{c\} \notin ji\text{-}F\alpha C(X)$.

We introduce the following definitions.

DEFINITION 4.6

A fuzzy bitopological space (X, τ_1, τ_2) is called an $ij\text{-}FT_k$ space if $ij\text{-}FGSC(X) = ij\text{-}F\psi^*C(X)$.

DEFINITION 4.7

A fuzzy bitopological space (X, τ_1, τ_2) is called an $ij\text{-}F\alpha T_k$ space if $ij\text{-}F\alpha GC(X) = ij\text{-}F\psi^*C(X)$.

DEFINITION 4.8

A fuzzy bitopological space (X, τ_1, τ_2) is called an $ij\text{-}FT_l$ space if $ij\text{-}FGSC(X) = ij\text{-}FG\alpha C(X)$.

DEFINITION 4.9

A fuzzy bitopological space (X, τ_1, τ_2) is called an $ij\text{-}F\alpha T_l$ space if $ij\text{-}F\alpha GC(X) = ij\text{-}FG\alpha C(X)$.

We show that the class of $ij\text{-}F\alpha T_k$ spaces properly contains the class of $ij\text{-}F\alpha T_e$ spaces and is properly contained in the class of $ij\text{-}F\alpha T_l$ spaces. We also show that the class of $ij\text{-}F\alpha T_k$ spaces is the dual of the class of $ij\text{-}FT_{1/5}^{\psi^*}$ spaces to the class of $ij\text{-}F\alpha T_e$ spaces. Moreover we prove that $ij\text{-}F\alpha T_k$ ness and $ij\text{-}FT_{1/5}^{\psi^*}$ ness are independent from each other.

THEOREM 4.10

Every $ij\text{-}F\alpha T_e$ space is an $ij\text{-}F\alpha T_k$ space.

PROOF.

Let (X, τ_1, τ_2) be an $ij\text{-}F\alpha T_e$ space. Let $A \in ij\text{-}F\alpha GC(X)$. Since (X, τ_1, τ_2) is an $ij\text{-}F\alpha T_e$ space, then $A \in ji\text{-}F\alpha C(X)$. Hence, by using Theorem 3, we have $A \in ij\text{-}F\psi^*C(X)$. Therefore (X, τ_1, τ_2) is an $ij\text{-}F\alpha T_k$ space.

The following example supports that the converse of the above theorem is not true in general.

EXAMPLE 4.11

Let X, τ_1 and τ_2 be as in the Example 4.2. Then (X, τ_1, τ_2) is an $ij\text{-}F\alpha T_k$ space but not an $ij\text{-}F\alpha T_e$ space since $\{a, c\} \in ij\text{-}F\alpha GC(X)$ but $\{a, c\} \notin ji\text{-}F\alpha C(X)$.

THEOREM 4.11

Every $ij\text{-}F\alpha T_k$ space is an $ij\text{-}F\alpha T_l$ space.

PROOF.

Let (X, τ_1, τ_2) be an $ij\text{-}F\alpha T_k$ space. Let $A \in ij\text{-}F\alpha GC(X)$. Since (X, τ_1, τ_2) is an $ij\text{-}F\alpha T_k$ space, then $A \in ij\text{-}F\psi^*C(X)$. Hence, by using Theorem 3.2, we have $A \in ji\text{-}FG\alpha C(X)$. Therefore (X, τ_1, τ_2) is an $ij\text{-}F\alpha T_l$ space.

The following example supports that the converse of the above theorem is not true in general.

EXAMPLE 4.12

Let X, τ_1 and τ_2 be as in the Example 4.1. Then (X, τ_1, τ_2) is an $ij\text{-}F\alpha T_l$ space but not an $ij\text{-}F\alpha T_k$ space since $\{b\} \in ij\text{-}F\alpha GC(X)$ but $\{b\} \notin ji\text{-}F\psi^*C(X)$.

THEOREM 4.12

A fuzzy bitopological space (X, τ_1, τ_2) is an $ij\text{-}F\alpha T_e$ space if and only if it is $ij\text{-}F\alpha T_k$ and $ij\text{-}FT_{1/5}^{\psi^*}$ space.

PROOF.

The necessity follows from the Theorems 4.9 and 4.10. For the sufficiency, suppose that (X, τ_1, τ_2) is both $ij\text{-}F\alpha T_k$ and $ij\text{-}FT_{1/5}^{\psi^*}$ space. Let $A \in ij\text{-}F\alpha GC(X)$. Since (X, τ_1, τ_2) is $ij\text{-}F\alpha T_k$ space, then $A \in ij\text{-}F\psi^*C(X)$. Since (X, τ_1, τ_2) is an $ij\text{-}FT_{1/5}^{\psi^*}$ space, then $A \in ji\text{-}F\alpha C(X)$. Thus (X, τ_1, τ_2) is an $ij\text{-}F\alpha T_e$ space.

REMARK 4.12

$ij\text{-}F\alpha T_k$ ness and $ij\text{-}FT_{1/5}^{\psi^*}$ ness are independent as it can be seen from the next two examples.

EXAMPLE 4.13

Let X, τ_1 and τ_2 be as in the Example 4.2. Then (X, τ_1, τ_2) is an $ij\text{-}F\alpha T_k$ space but not an $ij\text{-}FT_{1/5}^{\psi^*}$ space since $\{a, b\} \in ij\text{-}F\psi^* C(X)$ but $\{a, b\} \notin ji\text{-}F\alpha C(X)$.

EXAMPLE 4.14

Let X, τ_1 and τ_2 be as in the Example 4.1. Then (X, τ_1, τ_2) is an $ij\text{-}FT_{1/5}^{\psi^*}$ space since $\{b, c\} \in ij\text{-}F\alpha GC(X)$ but $\{b, c\} \notin ij\text{-}F\psi^* C(X)$.

DEFINITION 4.10

A fuzzy subset A of a fuzzy bitopological space (X, τ_1, τ_2) is called an ij-fuzzy ψ^*-open if its fuzzy complement is an ij-fuzzy ψ^*-closed of (X, τ_1, τ_2).

THEOREM 4.13

If (X, τ_1, τ_2) is an $ij\text{-}F\alpha T_k$ space, then for each $x \in X$, $\{x\}$ is either ij-fuzzy αg-closed or ij-fuzzy ψ^*-open.

PROOF.

Suppose that (X, τ_1, τ_2) is an $ij\text{-}F\alpha T_k$ space. Let $x \in X$ and assume that $\{x\} \notin ij\text{-}F\alpha GC(X)$. Then $\{x\} \notin ij\text{-}F\alpha C(X)$ since every ji-fuzzy α-closed set is an ij-fuzzy αg-closed set. So $X - \{x\} \notin ji\text{-}F\alpha O(X)$. Therefore X-$\{x\} \in ij\text{-}F\alpha GC(X)$ since X is the only ji-fuzzy α-open set which contains X-$\{x\}$. Since (X, τ_1, τ_2) is an $ij\text{-}F\alpha T_k$ space, then X-$\{x\} \in ij\text{-}F\psi^* C(X)$ or equivalently $\{x\} \in ij\text{-}F\psi^* O(X)$.

THEOREM 4.14

Every $ij\text{-}F\alpha T_k$ space is an $ij\text{-}\psi^*{}_{FT_{1/5}}$ space.

PROOF.

Let (X, τ_1, τ_2) be an $ij\text{-}F\alpha T_k$ space. Let $A \in ij\text{-}FG\alpha C(X)$, then $A \in ij\text{-}F\alpha GC(X)$. Since (X, τ_1, τ_2) is an $ij\text{-}F\alpha T_k$ space, then $A \in ij\text{-}F\psi^* C(X)$. Therefore (X, τ_1, τ_2) is an $ij\text{-}\psi^*{}_{FT_{1/5}}$ space.

The following example supports that the converse of the above theorem is not true in general.

EXAMPLE 4.15

Let X, τ_1 and τ_2 be as in the Example 4.8. Then (X, τ_1, τ_2) is an $ij\text{-}\psi^*{}_{FT_{1/5}}$ space but not an $ij\text{-}F\alpha T_k$ space since $\{c\} \in ij\text{-}F\alpha GC(X)$ but $\{c\} \notin ij\text{-}F\psi^* C(X)$.

We show that the class of $ij\text{-}FT_k$ spaces properly contains the class of $ij\text{-}FT_e$ spaces, and is properly contained in the class of $ij\text{-}F\alpha T_k$ spaces, the class of $ij\text{-}FT_l$ spaces, and the class of $ij\text{-}F\alpha T_l$ spaces.

THEOREM 4.15

Every $ij\text{-}FT_e$ space is an $ij\text{-}FT_k$ space.

PROOF.

Let (X, τ_1, τ_2) be an $ij\text{-}FT_e$ space. Let $A \in ij\text{-}FGSC(X)$. Since (X, τ_1, τ_2) is an $ij\text{-}FT_e$ space, then $A \in ji\text{-}F\alpha C(X)$. Hence, by using Theorem 3.1, we have $A \in ij\text{-}F\psi^* C(X)$. Therefore (X, τ_1, τ_2) is an $ij\text{-}FT_k$ space.

The following example supports that the converse of the above theorem is not true in general.

EXAMPLE 4.16

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Let X, τ_1 and τ_2 be as in the Example 4.2. Then (X, τ_1, τ_2) is an $ij\text{-}FT_k$ space but not an $ij\text{-}FT_e$ space since $\{a, c\} \in ij\text{-}FGSC(X)$ but $\{a, c\} \notin ji\text{-}F\alpha C(X)$.

THEOREM 4.16

Every $ij\text{-}FT_k$ space is an $ij\text{-}F\alpha T_k$ space.

PROOF.

Let (X, τ_1, τ_2) be an $ij\text{-}FT_k$ space. Let $A \in ij\text{-}F\alpha GC(X)$, then $A \in ij\text{-}FGSC(X)$. Since (X, τ_1, τ_2) is an $ij\text{-}FT_k$ space, then $A \in ij\text{-}F\psi^*C(X)$. Therefore (X, τ_1, τ_2) is an $ij\text{-}F\alpha T_k$ space.

The converse of the above theorem is not true as it can be seen from the following example.

EXAMPLE 4.17

Let X, τ_1 and τ_2 be as in the Example 4.5. Then (X, τ_1, τ_2) is an $ij\text{-}F\alpha T_k$ space but not an $ij\text{-}FT_k$ space since $\{b\} \in ij\text{-}FGSC(X)$ but $\{b\} \notin ij\text{-}F\psi^*C(X)$.

THEOREM 4.17

Every $ij\text{-}FT_k$ space is an $ij\text{-}FT_l$ space.

PROOF.

Let (X, τ_1, τ_2) be an $ij\text{-}FT_k$ space. Let $A \in ij\text{-}FGSC(X)$. Since (X, τ_1, τ_2) is an $ij\text{-}FT_k$ space, then $A \in ij\text{-}F\psi^*GC(X)$. Hence, by using Theorem 3.2, we have $A \in ij\text{-}FG\alpha C(X)$. Therefore (X, τ_1, τ_2) is an $ij\text{-}FT_l$ space.

The converse of the above theorem is not true as it can be seen from the following example.

EXAMPLE 4.18

Let $X = \{a, b, c\}, \tau_1 = \{X, \phi, \{a, b\}\}$ and $\tau_2 = \{X, \phi, \{a, c\}\}$. Then (X, τ_1, τ_2) is an $ij - FT_l$ space but not an $ij\text{-}FT_k$ space since $\{c\} \in ij\text{-}FGSC(X)$ but $\{c\} \notin ij\text{-}F\psi^*C(X)$.

Next we prove that the dual of the class of $ij\text{-}FT_l$ spaces to the class of $ij\text{-}FT_k$ spaces is the class of $ij\text{-}F\alpha T_k$ spaces.

THEOREM 4.18

A fuzzy bitopological space (X, τ_1, τ_2) is an $ij\text{-}FT_k$ space if and only if it is $ij\text{-}F\alpha T_k$ and $ij\text{-}FT_l$ space.

PROOF.

The necessity follows from the Theorem 4.16 and 4.17. For the sufficiency, suppose that (X, τ_1, τ_2) is both $ij\text{-}F\alpha T_k$ and $ij\text{-}FT_l$ space. Let $A \in ij\text{-}FGSC(X)$. Since (X, τ_1, τ_2) is an $ij\text{-}FT_l$ space, then $A \in ij\text{-}FG\alpha C(X)$. Then $A \in ij\text{-}F\alpha GC(X)$. Since (X, τ_1, τ_2) is an $ij\text{-}F\alpha T_k$ space, then $A \in ij\text{-}F\psi^*C(X)$. Therefore (X, τ_1, τ_2) is an $ij\text{-}FT_k$ space.

THEOREM 4.19

A fuzzy bitopological space (X, τ_1, τ_2) is an $ij\text{-}FT_e$ space if and only if it is $ij\text{-}FT_k$ and $ij\text{-}FT_{1/5}^{\psi^*}$ space.

PROOF.

The necessity follows from the Theorems 4.8 and 4.15. For the sufficiency, suppose that (X, τ_1, τ_2) is both $ij\text{-}FT_k$ and $ij\text{-}FT_{1/5}^{\psi^*}$ space.

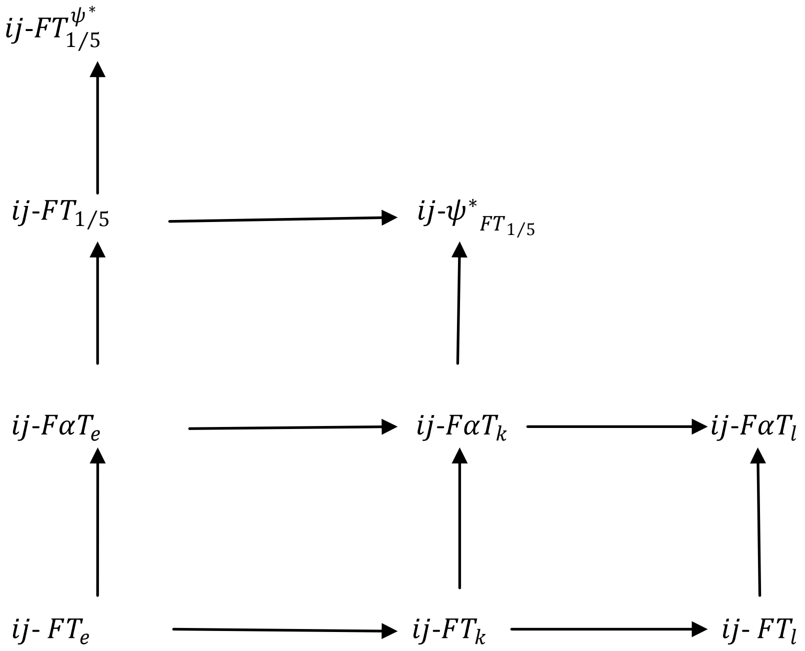

Diagram 2

Let $A \in ij$-$FGSC(X)$. Since (X, τ_1, τ_2) is an ij-FT_k space, then $A \in ij$-$F\psi^*C(X)$. Since (X, τ_1, τ_2) is an ij-$FT_{1/5}^{\psi^*}$ space, the $A \in ji$-$F\alpha C(X)$. Therefore (X, τ_1, τ_2) is an ij-FT_e space.

The following diagram shows the relationships between the separation axioms discussed in this section (Diagram 2).

EXAMPLE 4.19

Let $X = \{a, b, c\}$ and δ_1, δ_2, δ_3 be fuzzy sets of X defined as follows

$\delta_1 = \{(0.5, 0.5, 0.5), (0.2, 0, 0), (0.8, 1, 1)\}$

$\delta_2 = \{(0.5, 0.5, 0.5), (0, 0.1, 0), (1, 0.9, 1)\}$

Clearly,

$\tau_1 = \{0,1, \delta_1, \delta_2, \delta_1 \vee \delta_2\}$ and

$\tau_2 = \{0,1, \delta_3\}$ are fuzzy topologies on X.

Let $f : (X, \tau_1) \to (X, \tau_2)$ be defined by $f(x) = x$ for each $x \in X$.

(arrows 1,2) $\delta_2 \in 12$-$F\alpha T_e$ $\wedge$ 12-FT_k space but not an 12- FT_e space. Since $\{b\} \in 12$- $FGSC(X)$ but $\{b\} \notin 21$- $F\alpha C(X)$.

(arrow 3,6) $\delta_1 \in 12$-$F\alpha T_k$ $\wedge$ 12-FT_l space but an 12- FT_k space. Since $\{c\} \in 12$- $FGSC(X)$ but $\{c\} \notin 12$- $F\psi^*C(X)$.

(arrow 8,4) $\delta_2 \in 12$-$FT_{1/5}$ $\wedge$ 12-$F\alpha T_k$ space but an 12- $F\alpha T_e$ space. Since $\{a, c\} \in 12$-$F\alpha GC(X)$ but an $\{a, c\} \notin 21$- $F\alpha C(X)$.

(arrow 5) $\delta_2 \in 12$-$F\alpha T_l$ space but an 12- FT_e space. Since $\{c\} \in 12$- $F\alpha GC(X)$ but $\{c\} \notin 21$-$F\alpha C(X)$.

(arrow 7) $\delta_2 \in 12$-$F\alpha T_l$ space but an 12- FT_e space. Since $\{c\} \in 12$- $FGSC(X)$ but $\{c\} \notin 21$-$F\alpha C(X)$.

(arrow 9) $\delta_2 \in 12$-${\psi^*}_{FT_{1/5}}$ space but an 12-$F\alpha T_k$ space. Since $\{c\} \in 12$- $F\alpha GC(X)$ but $\{c\} \notin 12$- $F\psi^*C(X)$.

(arrow 10) $\delta_2 \in 12$-${\psi^*}_{FT_{1/5}}$space but an 12-$FT_{1/5}$ space. Since $\{b, c\} \in 12$- $FG\alpha C(X)$ but $\{b, c\} \notin 21$- $F\alpha C(X)$.

(arrow 11) $\delta_1 \in 12$-$FT_{1/5}^{\psi^*}$ space but 12- $FT_{1/5}$ space. Since $\{a, b\} \in 12$- $FG\alpha C(X)$ but $\{a, b\} \notin 21$- $F\alpha C(X)$.

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ij-FUZZY ψ^*-CONTINUOUS AND ij-FUZZY ψ^*-IRRESOLUTE FUNCTIONS

We introduce the following definition.

DEFINITION 5.1

A function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is called ij-fuzzy ψ^*-continuous if $\forall\, V\in j\text{-}FC(Y), f^{-1}(V)\in ij\text{-}F\psi^*C(X)$.

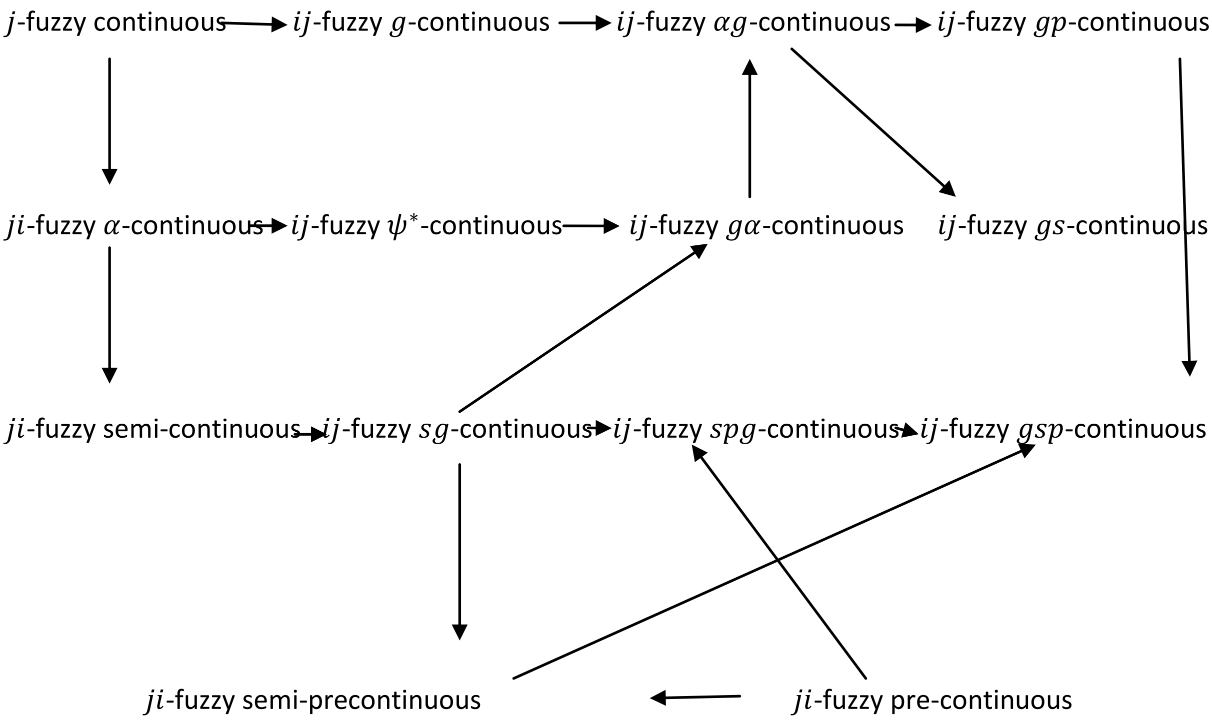

DIAGRAM 3

The following diagram shows the relationships of ij-fuzzy ψ^*-continuous functions with some other functions discussed in this section (Diagram 3).

EXAMPLE 5.1

Let $X=\{a,b,c\}$. The fuzzy sets λ,μ,v,A,B are defined on X as follows,

$\lambda:X\to[0,1]$ is defined as $\lambda(a)=0.4,\lambda(b)=0.5,\lambda(c)=0.5$.

$\mu:X\to[0,1]$ is defined as $\mu(a)=0.4,\mu(b)=0.4,\mu(c)=0.5$.

$v:X\to[0,1]$ is defined as $v(a)=0.3,v(b)=0.5,v(c)=0.2$.

$A:X\to[0,1]$ is defined as $A(a)=0.4,A(b)=0.3,A(c)=0.5$.

$B:X\to[0,1]$ is defined as $B(a)=0.5,B(b)=0.4,B(c)=0.6$

Then $\tau_1=\{0,1,\lambda,\mu,v\}$ and

$\tau_2=\{0,1,A,B\}$ are fuzzy topologies on X.

Let $f:(X,\tau_1)\to(X,\tau_2)$ be defined by $f(a)=a$ for each $x\in X$.

(arrows 1,5) If f is defined by $f(a)=b$, $f(b)=c$ and $f(c)=a$. We have f is 12-Fg-continuous, but it is not 1-Fuzzy continuous. Since there exist $\{b\}\in 1\text{-}FC(X)$ but $f^{-1}(\{b\})=\{a\}\notin 1\text{-}FC(X)$. Also, f is 12-$Fg\alpha$-continuous but it is not 2-Fuzzy continuous. Since there exist $\{b,a\}\in 2\text{-}FC(X)$ such that $f^{-1}(\{b,a\})=\{a,c\}\notin 2\text{-}FC(X)$.

(arrows 4,6) If f is defined by $f(a)=a,f(b)=c$ and $f(c)=b$. We have f is 12-Fuzzy ψ^*-continuous, but it is not 21-Fuzzy 2-continuous. Since there exist $\{a\}\in 2\text{-}FC(X)$ but

$f^{-1}(\{a\}) = \{c\} \notin 12\text{-}FGC(X)$. Also, f is not 2-Fuzzy continuous. Since there exist $\{b, a\} \in 2\text{-}FC(X)$ such that $f^{-1}(\{b, a\}) = \{a, b\} \notin 12\text{-}FPC(X)$.

(arrows 7,15) If f is defined by $f(a) = c, f(b) = a$ and $f(c) = b$. We have f is $12\text{-}Fspg$-continuous ∧ 21-Fuzzy semi-Pre Continuous, but it is not $12\text{-}Fsg$-continuous. Since there exist $\{b\} \in 21\text{-}FC(X)$ such that $f^{-1}(\{b\}) = \{b, c\} \notin 12\text{-}FGPC(X)$.

(arrow 2) If f is defined by $f(a) = f(b) = c$ and $f(c) = a$. We have f is $12\text{-}F\alpha g$-continuous, but it is not 2-Fuzzy continuous. Since there exist $\{c\} \in 2\text{-}FC(X)$ but $f^{-1}(\{c\}) = \{a\} \notin 2\text{-}FGC(X)$.

(arrow 3) If f is defined by $f(c) = f(a) = b$ and $f(b) = a$. We have f is $12\text{-}Fgp$-continuous, but it is not 2-Fuzzy continuous. Since there exist $\{b\} \in 2\text{-}FPC(X)$ but $f^{-1}(\{b\}) = \{a\} \notin 2\text{-}FC(X)$.

(arrow 8) If f is defined by $f(a) = b$, and $f(b) = f(c) = a$. We have f is $12\text{-}Fgsp$-continuous, but it is not 21-Fuzzy semi-continuous.

(arrow 9) If f is defined by $f(a) = c, f(b) = a$ and $f(c) = b$. We have f is 21-Fuzzy α-continuous, but it is not 1-Fuzzy continuous.

(arrow 10) If f is defined by $f(a) = f(b) = c$ and $f(c) = b$. We have f is $12\text{-}F\alpha g$-continuous, but it is not 2-Fuzzy continuous. Since there exist $\{c\} \in 21\text{-}F\alpha C(X)$ such that $f^{-1}(\{c\}) = \{a, b\} \notin 12\text{-}FGC(X)$.

(arrow 11) If f is defined by $f(b) = f(c) = b$ and $f(a) = c$. We have f is $12\text{-}Fgs$-continuous, but it is not 21-Fuzzy α-continuous.

(arrow 12) If f is defined by $f(a) = f(c) = b$ and $f(b) = a$. We have f is 21-Fuzzy semi continuous, but it is not 21-Fuzzy α-continuous.

(arrow 13) If f is defined by $f(a) = f(b) = c$ and $f(c) = b$. We have f is $12\text{-}Fg\alpha$-continuous, but it is not 2-Fuzzy continuous.

(arrow 14) If f is defined by $f(b) = f(c) = b$ and $f(a) = c$. We have f is $12\text{-}Fgsp$-continuous. Since there exist $\{b\} \in 2\text{-}FgC(X)$ but it is not $f^{-1}(\{b\}) = \{c\} \notin 12\text{-}FC(X)$.

(arrow 16) If f is defined by $f(a) = f(b) = f(c) = a$. We have f is $12\text{-}Fgsp$-continuous, but it is not 21-Fuzzy semi continuous.

(arrow 17) If f is defined by $f(c) = b, f(b) = a$ and $f(a) = c$. We have f is 12-Fuzzy semi pre-continuous, but it is not 21-Fuzzy semi-continuous.

(arrow 18) If f is defined by $f(a) = b, f(b) = c$ and $f(c) = a$. We have f is $12\text{-}Fspg$-continuous, but it is not 12-Fuzzy α-continuous. Since there exist $\{b\} \in 1\text{-}FC(X)$ such that $f^{-1}(\{b\}) = \{a\} \notin 1\text{-}FC(X)$.

THEOREM 5.1

Every ji-fuzzy α-continuous function is ij-fuzzy ψ^*-continuous.

The following example supports that the converse of the above theorem is not true in general.

EXAMPLE 5.2

Let $X = \{a, b, c\}$ and $Y = \{\alpha, \beta, \gamma\}$

Define fuzzy sets λ_1, λ_2 and μ_1 as follows

$\lambda_1(a) = 0.4, \lambda_1(b) = 0.3, \lambda_1(c) = 0.2$

$\lambda_2(a) = \mu_1(\alpha) = 0.6, \quad \lambda_2(b) = \mu_1(\beta) = 0.7, \lambda_2(c) = \mu_1(\gamma) = 0.8$

Let τ_1, τ_2 and σ_1, σ_2 be defined as follows

$$\tau_1(\lambda) = \begin{cases} 1 & , if\ \lambda = 0\ or\ 1 \\ 1/2 & , if\ \lambda = \lambda_1 \\ 0 & , otherwise \end{cases}$$

$$\tau_2(\lambda) = \begin{cases} 0 & , if\ \lambda = 0\ or\ 1 \\ 1/2 & , if\ \lambda = \lambda_1 \\ 1 & , otherwise \end{cases}$$

$$\sigma_1(\mu) = \begin{cases} 1 & , if\ \mu = 0\ or\ 1 \\ 1/2 & , if\ \mu = \mu_1 \\ 0 & , otherwise \end{cases}$$

$$\sigma_2(\mu) = \begin{cases} 0 & , if\ \mu = 0\ or\ 1 \\ 1/2 & , if\ \mu = \mu_1 \\ 1 & , otherwise \end{cases}$$

Then the function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is defined by

$f(a) = \alpha, f(b) = \beta, f(c) = \gamma.$

Then f is not 21-fuzzy α-continuous function.

Since $\mu_1 \in$ 2-$FC(Y)$ but $f^{-1}(\mu_1) = \lambda_2 \notin$ 21-$F\alpha C(X)$.

However f is 12-Fuzzy ψ^*-continuous function.

THEOREM 5.2

Every ij-fuzzy ψ^*-continuous function is ij-fuzzy $g\alpha$-continuous.

The following example supports that the converse of the above theorem is not true in general.

EXAMPLE 5.3

Let $X = Y = \{a, b, c\}$

Define fuzzy sets $\lambda, \delta, \beta : X = Y \to [0, 1]$ by the equation

$\lambda(a) = 0.5, \lambda(b) = 0, \lambda(c) = 0$

$\delta(a) = 0, \delta(b) = 0.6, \delta(c) = 0$ and

$\beta(a) = 0.6, \beta(b) = 0.6, \beta(c) = 1$

Then $\tau_1 = \{1, 0, \lambda, \beta\}$ and

$\tau_2 = \{1, 0\ \delta\}$ are fuzzy topologies on X and Y.

Let δ be the non fuzzy open set in (X, τ_1).

Then

$\tau(\delta) = \{1, 0, \lambda, \delta, \lambda \vee \delta\}.$

Let $\lambda_1(a) = 0.3$, $\lambda_1(b) = 0$, $\lambda_1(c) = 0$ be the fuzzy subset in (X, τ_1).

If $f : (X, \tau_1) \to (Y, \tau_2)$ be defined by

$f(a) = a, f(b) = b, f(c) = c,$

then f is not 12-Fuzzy ψ^*-continuous function.

Since $\delta \in$ 2-$FC(Y)$ but $f^{-1}(\delta) = \beta \notin$ 12-$F\psi^* C(X)$.

However f is 12-$Fg\alpha$-continuous function.

THEOREM 5.3

If $f_1 : (X_1, \tau_1, \tau_2) \to (Y_1, \sigma_1, \sigma_2)$ and $f_1 : (X_2, \tau_1^*, \tau_2^*) \to (Y_1, \sigma_1^*, \sigma_2^*)$ be two ij-fuzzy ψ^*-continuous functions. Then the function $f : (X_1 \times X_2, \tau_1 \times \tau_1^*, \tau_2 \times \tau_2^*) \to (Y_1 \times Y_2, \sigma_1 \times \sigma_1^*, \sigma_2 \times \sigma_2^*)$ defined by $f(x_1, x_2) = \big(f(x_1), f(x_2)\big)$ is ij-fuzzy ψ^*-continuous.

PROOF.

Let $V_1 \in j$-$FO(Y_1)$ and $V_2 \in j$-$FO(Y_2)$. Since f_1 and f_2 are two ij-fuzzy ψ^*-continuous, then $f^{-1}(V_1) \in ij$-$F\psi^* O(X_1)$ and $f^{-1}(V_2) \in ij$-$F\psi^* O(X_2)$. Hence, by using Theorem 3.5, we have $f^{-1}(V_1) \times f^{-1}(V_2) \in ij$-$F\psi^* O(X_1 \times X_1)$.

We introduce the following definition.

DEFINITION 5.2

A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called ij-fuzzy ψ^*-irresolute if $\forall\, V \in ij$-$F\psi^* C(Y), f^{-1}(V) \in ij$-$F\psi^* C(X)$.

THEOREM 5.4

Every ij-fuzzy ψ^*-irresolute function is ij-fuzzy ψ^*-continuous.

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The following example supports that the converse of the above theorem is not true in general.

EXAMPLE 5.4

Let $X = Y = \{a, b, c\}$.

Define fuzzy sets $\lambda, \delta_1, \delta_2 : X \to [0, 1]$ by the equation

$\lambda(a) = 0.4, \lambda(b) = 0, \lambda(c) = 1$

$\delta_1(a) = 0$, $\delta_1(b) = 0.5$, $\delta_1(c) = 0$ and

$\delta_2(a) = 0$, $\delta_2(b) = 0$, $\delta_2(c) = 0.6$

And $\gamma : Y \to [0,1]$ defined by

$\gamma(a) = 1, \gamma(b) = 0.5, \gamma(c) = 0$

Then $\tau_1 = \{1, 0, \lambda\}$ and

$\tau_2 = \{1, 0, \gamma\}$ is a fuzzy topologies on X and Y.

Let δ_1 be the non fuzzy open set in (X, τ_1), then $\tau_1(\delta_1) = \{1, 0, \lambda, \delta_1, \lambda \vee \delta_1\}$ and δ_2 be the non fuzzy open set in (Y, τ_2), then $\tau_2(\delta_2) = \{1, 0, \gamma, \delta_2, \gamma \vee \delta_2\}$.

Let $f : (X, \tau_1) \to (Y, \tau_2)$ be defined by

$f(a) = b, f(b) = a, f(c) = c$

Then f is not 12-Fuzzy ψ^*-irresolute function.

Since $\delta_1 \in 12\text{-}F\psi^*C(Y)$ but $f^{-1}(\delta_1) = \gamma \notin 12\text{-}F\psi^*C(X)$.

However f is 12-Fuzzy ψ^*-continuous function.

THEOREM 5.5

Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ and $g : (Y, \sigma_1, \sigma_2) \to (Z, \eta_1, \eta_2)$ be any two functions. Then

(1) $g \circ f$ is ij-fuzzy ψ^*-continuous if g is j-fuzzy continuous and f is ij-fuzzy ψ^*-continuous.

(2) $g \circ f$ is ij-fuzzy ψ^*-irresolute if both f and g are ij-fuzzy ψ^*-irresolute.

(3) $g \circ f$ is ij-fuzzy ψ^*-continuous if g is ij-fuzzy ψ^*-continuous and f is ij-fuzzy ψ^*-irresoloute.

PROOF.

Let $V \in j\text{-}FC(Z)$, since g is j-fuzzy continuous, then $g^{-1}(V) \in j\text{-}FC(Y)$. Since f is ij-fuzzy ψ^*-continuous, then we have $f^{-1}\left(g^{-1}(V)\right) \in ij\text{-}F\psi^*C(X)$. Consequently, $g \circ f$ is ij-fuzzy ψ^*-continuous.

(2)- (3) Similarly.

THEOREM 5.6

Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be an ij-fuzzy ψ^*-continuous function. If (X, τ_1, τ_2) is $ij\text{-}FT_{1/5}^{\psi^*}$ space, then f is ji-fuzzy α-continuous function.

PROOF.

Let $V \in j\text{-}FC(Y)$. Since f is ij-fuzzy ψ^*-continuous, then $f^{-1}(V) \in ij\text{-}F\psi^*C(X)$. Since (X, τ_1, τ_2) is an $ij\text{-}FT_{1/5}^{\psi^*}$ space, then $f^{-1}(V) \in ji\text{-}F\alpha C(X)$. Consequently, f is ji-fuzzy α-continuous.

THEOREM 5.7

Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be an ij-fuzzy αg-continuous function. If (X, τ_1, τ_2) is an $ij\text{-}F\alpha T_k$ space, then f is ij-fuzzy ψ^*-continuous.

PROOF.

Let $V \in j\text{-}FC(Y)$. Since f is an ij-fuzzy αg-contiuous function, thus $f^{-1}(V) \in ij\text{-}F\alpha GC(X)$. Since (X, τ_1, τ_2) is an $ij\text{-}F\alpha T_k$ space, then $f^{-1}(V) \in ij\text{-}F\psi^*C(X)$. Consequently, f is ij-fuzzy ψ^*-continuous.

THEOREM 5.8

Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be an ij-fuzzy $g\alpha$-continuous function. If (X, τ_1, τ_2) is $ij\text{-}{\psi^*}_{FT_{1/5}}$ space, then f is ij-fuzzy ψ^*-continuous.

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PROOF.

Let $V \in j\text{-}FC(Y)$. Since f is an ij-fuzzy $g\alpha$-continuous function, thus $f^{-1}(V) \in ij\text{-}FG\alpha C(X)$. Since (X, τ_1, τ_2) is an $ij\text{-}\psi^*{}_{FT_{1/5}}$ space, then $f^{-1}(V) \in ij\text{-}F\psi^*C(X)$. Consequently, f is ij-fuzzy ψ^*-continuous.

THEOREM 5.9

Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be an ij-fuzzy gs-continuous function. If (X, τ_1, τ_2) is $ij\text{-}FT_k$ space, then f is ij-fuzzy ψ^*-continuous.

PROOF.

Let $V \in j\text{-}FC(Y)$. Since f is an ij-fuzzy gs-continuous function, thus $f^{-1}(V) \in ij\text{-}FGSC(X)$. Since (X, τ_1, τ_2) is an $ij\text{-}FT_k$ space, then $f^{-1}(V) \in ij\text{-}F\psi^*C(X)$. Consequently, f is ij-fuzzy ψ^*-continuous.

THEOREM 5.10

Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be onto, ij-fuzzy ψ^*-irresolute and ji-fuzzy α-closed. If (X, τ_1, τ_2) is $ij\text{-}FT_{1/5}^{\psi^*}$ space, then (Y, σ_1, σ_2) is also an $ij\text{-}FT_{1/5}^{\psi^*}$ space.

PROOF.

Let $V \in ij\text{-}F\psi^*C(Y)$. Since f is ij-fuzzy ψ^*-irresolute, then $f^{-1}(V) \in ij\text{-}F\psi^*C(X)$. Since (X, τ_1, τ_2) is $ij\text{-}FT_{1/5}^{\psi^*}$ space, then $f^{-1}(V) \in ji\text{-}F\alpha C(X)$. Since f is ji-fuzzy α-closed and onto. Then we have $V \in ji\text{-}F\alpha C(Y)$. Therefore (Y, σ_1, σ_2) is also an $ij\text{-}FT_{1/5}^{\psi^*}$ space.

We introduce the following definition.

DEFINITION 5.3

A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called an ij-fuzzy pre-ψ^*-closed if $A \in ij\text{-}F\psi^*C(X), f(A) \in ij\text{-}F\psi^*C(Y)$.

THEOREM 5.11

Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be onto, ij-fuzzy $g\alpha$-irresolute and ij-fuzzy pre-ψ^*-closed. If (X, τ_1, τ_2) is $ij\text{-}\psi^*{}_{FT_{1/5}}$ space, then (Y, σ_1, σ_2) is also an $ij\text{-}\psi^*{}_{FT_{1/5}}$ space.

PROOF.

Let $V \in ij\text{-}FG\alpha C(Y)$. Since f is ij-fuzzy $g\alpha$-irresolute, then $f^{-1}(V) \in ij\text{-}FG\alpha C(X)$. Since (X, τ_1, τ_2) is an $ij\text{-}\psi^*{}_{FT_{1/5}}$ space. Since f is ij-fuzzy pre-ψ^*-closed and onto. Then we have $f\big(f^{-1}(V)\big) = V \in ij\text{-}F\psi^*C(Y)$. Therefore (Y, σ_1, σ_2) is also an $ij\text{-}\psi^*{}_{FT_{1/5}}$ space.

THEOREM 5.12

Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be onto, ij-fuzzy αg-irresolute and ij-fuzzy pre-ψ^*-closed. If (X, τ_1, τ_2) is an $ij\text{-}F\alpha T_k$ space, then (Y, σ_1, σ_2) is also an $ij\text{-}F\alpha T_k$ space.

PROOF.

Let $V \in ij\text{-}F\alpha GC(Y)$. Since f is ij-fuzzy αg-irresolute, then $f^{-1}(V) \in ij\text{-}F\alpha GC(X)$. Since (X, τ_1, τ_2) is an $ij\text{-}F\alpha T_k$ space, then $f^{-1}(V) \in ij\text{-}F\psi^*C(X)$. Since f is ij-fuzzy pre-ψ^*-closed and onto. Then we have $f\big(f^{-1}(V)\big) = V \in ij\text{-}F\psi^*C(Y)$. Therefore, (Y, σ_1, σ_2) is also an $ij\text{-}F\alpha T_k$ space.

THEOREM 5.13

Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be onto, ij-fuzzy gs-irresolute and ij-fuzzy pre-ψ^*-closed. If (X, τ_1, τ_2) is an $ij\text{-}FT_k$ space, then (Y, σ_1, σ_2) is also an $ij\text{-}FT_k$ space.

PROOF.

Let $V \in ij\text{-}FGSC(Y)$. Since f is ij-fuzzy gs-irresolute, then $f^{-1}(V) \in ij\text{-}FGSC(X)$. Since (X, τ_1, τ_2) is an $ij\text{-}FT_k$ space, then $f^{-1}(V) \in ij\text{-}F\psi^*C(X)$. Since f is ij-fuzzy pre- ψ^*-closed and onto. Then we have $f\big(f^{-1}(V)\big) = V \in ij\text{-}F\psi^*C(Y)$. Therefore (Y, σ_1, σ_2) is also an $ij\text{-}FT_k$ space.

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