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# FUZZY SOFT ALPHA-OPEN SETS AND FUZZY SOFT ALPHA-CONTINUOUS FUNCTIONS

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## INTRODUCTION

In real life situation, the problems in Economics, Engineering, Social sciences, Medical science etc. do not always involve crisp data. So, we cannot successfully use the traditional classical methods because of various types of uncertainities presented in these problems. To exceed these uncertainities, some kind of theories were given like theory of fuzzy set, intuitionistic fuzzy set, rough set, bipolar fuzzy set, i.e. which we can use as mathematical tools for dealings with uncertainities. But, all these theories have their inherent difficulities. The reason for these difficulties Molodtsov [1] initiated the concept of soft set theory as a new mathematical tool for dealing with uncertainities which is free from the above difficulties. In [1], Molodtsov successfully applied the soft set theory in several directions, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, Probability, theory of measurement, and so on. After presentation of the operations of soft sets [8], the properties and applications of soft set theory have been studied increasingily [7, 2]. Xiao et al [4] and Pei and Miao [5] discussed the relationship between soft sets and information systems. They showed that soft sets are a class of special information systems. In recent years, many interesting applications of soft theory have been expanded by embedding the ideas of fuzzy sets [2,8,9]. To develop soft set theory, the operations of the soft sets are redefined and a uni-intdecision making method was constructed by using these new operations [11].

Recently, 2011 Shabir and Naz[3] initiated the study of soft topological spaces. They defined soft topology on the collection $\tau$ of soft sets over $X$. Consequently, they defined basic notions of soft topological spaces such as open soft and closed soft sets, soft subspace, soft closure, soft neighborhood of a point, Soft separation axioms, Soft regular spaces and soft normal spaces and established their several properties. Min investigate some properties of these soft separation axioms. In [6], Kandil et al. introduced the notion of soft semi separation axioms. In particular they study the properties of the soft semi regular spaces and soft semi normal spaces. The notion of soft ideal was initiated for the first time by Kandil et al[6]. They also introduced the concept of soft local function. These concepts are discussed with a view to find new soft topologies from the original one, called soft topological spaces with soft ideal $(X, \tau, E, \tilde{I})$. Applications to various fields were further investigated by Kandil et al. The notion of b-open soft sets was initiated for the first time by El-Sheikh and Abd El-latif [12]. Maji et al [9] initiated the study involving bith fuzzy sets and soft sets.

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In [8] the notion of fuzzy soft set was introduced as a fuzzy generalization of soft sets and some basic properties of fuzzy soft sets are discussed in detail. Then, many Scientists such X. Yang et al. improved the concept of fuzziness of soft sets in karl et al. defined the notion of a mapping on classes of fuzzy soft sets, which is fundamental important in fuzzy soft set theory, to improve this work and they studied properties of fuzzy soft images and fuzzy soft inverse images of fuzzy soft sets.

Chang introduced the concept of fuzzy topology on a set X by axiomatizing a collection $\mathfrak{T}$ of fuzzy subset of X. Danay et al introduce the definition of fuzzy soft topology over a subset of the initial universe set while Roy and Samanta gave the definition of fuzzy soft topology over the initial universe set. Some fuzzy soft topological properties based on fuzzy pre (resp. semi, $\beta -$) open soft sets, were introduced in [1, 3,].

In the present paper, We introduce some new concepts in fuzzy soft topological spaces such as fuzzy soft $\alpha$- open sets, soft $\alpha$-closed sets and soft $\alpha$-continuous functions. We also study relationship between fuzzy soft continuity [6], fuzzy soft semi-continuity [7], and fuzzy soft $\alpha$-continuity of functions defined on fuzzy soft topological spaces. With the help of counter examples, We show the non-coincidence of these various types of mappings.

## PRELIMINARIES

### Definition 1 (see [1])

Let X be an initial universe and let E be a set of parameters. Let P(X) denote the power set of X and let A be a nonempty subset of E. A pair $(F,A)$ is called a fuzzy soft set over X, where F is a mapping given by $F : A \to P(X)$. In other words, a fuzzy soft set over X is a parameterized family of subsets of the universe X. For $\varepsilon \epsilon A, F(\varepsilon)$ may be considered as the set of $\varepsilon -$approximate elements of the fuzzy soft set $(F,A)$.

### Definition 2 (see [8])

A soft set $(F,A)$ over X is called a fuzzy null soft set, denoted by $\Phi$; if $e \epsilon A, F(e) = \emptyset$

### Definition 3 (see [8])

A soft set $(F,A)$ over X is called an fuzzy absolute soft set, denoted by $\tilde{A}$; if $e \epsilon A, F(e) = X$.

### Definition 4 (see [8])

The union of two fuzzy soft sets of $(F,A)$ and $(G,B)$ over the common universe X is the fuzzy soft set $(H,C)$, where $C = A \tilde{\sqcup} B$ and , for all $e \in C, H(e) = \begin{cases} F(e), if\ e \in A - B, \\ G(e), if\ e \in B - A \\ F(e) \tilde{\sqcup} G(e), if\ e \in A \tilde{\sqcap} B \end{cases}$

We write $(F,A) \tilde{\sqcup} (G,B) = (H,C)$.

### Definition 5 (see [8])

The intersection $(H,C)$ of two fuzzy soft sets $(F,A)$ and $(G,B)$ over a common universe X, denoted $(F,A) \tilde{\sqcap} (G,B)$, is defined as $C = A \tilde{\sqcap} B$ and $H(e) = F(e) \tilde{\sqcap} G(e)$ for all $e \in C$.

### Definition 6 (see [8])

Let $(F,A)$ and $(G,B)$ be two fuzzy soft sets over a common universe X. $(F,A) \tilde{\sqsubseteq} (G,B)$, if $A \sqsubset B$, and $H(e) = F(e) \sqsubset G(e)$, for all $e \in A$

### Definition 7 (see [2])

Let $\tau$ be the collection of fuzzy soft sets over X; then $\tau$ is said to be a fuzzy soft topology on X if it satisfies the following axioms:

1) $\varphi$ and $\bar{X}$ belong to $\tau$,
2) The union of any number of fuzzy soft sets in $\tau$ belongs to $\tau$,
3) The intersection of any two fuzzy soft sets in $\tau$ belongs to $\tau$.

The triplet $(X,\tau,E)$ is called a fuzzy soft topological space over X. Let $(X,\tau,E)$ be a fuzzy soft topological space over X; then the members of $\tau$ are said to be fuzzy soft sets in X. The relative complement of a fuzzy soft set $(F, A)$ is denoted by $(F, A)^C$ and is defined by $(F, A)^C = (F^c, A)$, where $F^c : A \to P(X)$ is a mapping given by $F^c(e) = X - F(e)$ for all $e \in A$. Let $(X, \tau, E)$ be a fuzzy soft topological space over X. A fuzzy soft set $(F, E)$ over X is said to be a fuzzy soft closed set in X if its relative complement $(F, E)^C$ belongs to $\tau$. If $(X, \tau, E)$ is a fuzzy soft topological space with $\tau = \{\Phi, \tilde{X}\}$, then $\tau$ is called the fuzzy soft indiscrete topology on X and $(X, \tau, E)$ is said to be a fuzzy soft indiscrete topological space. If $(X, \tau, E)$ is a fuzzy soft topological space with $\tau$ is the collection of all fuzzy soft sets which can be defined over X and $(X, \tau, E)$ is said to be a fuzzy soft discrete topological space.

### Definition 8

Let $(X, \tau, E)$ be a fuzzy soft topological space over X and let $(A, E)$ be a fuzzy soft set over X.

(1) [3] The fuzzy soft interior of $(A, E)$ is the fuzzy soft set

$$\text{int}(A, E) = \sqcup \left\{ \begin{matrix} (O, E): (O, E) \; which\ is\ fuzzy\ soft\ open\ and \\ (O, E) \tilde{\sqsubseteq} \; (A, E) \end{matrix} \right\}.$$

(2) [2] The fuzzy soft closure of $(A, E)$ is the fuzzy soft set

$$\text{cl}(A, E) = \sqcap \left\{ \begin{matrix} (F, E): (F, E) \; which\ is\ fuzzy\ soft\ closed\ and \\ (A, E) \tilde{\sqsubseteq} \; (F, E) \end{matrix} \right\}.$$

Clearly cl$(A, E)$ is the smallest soft closed set over X which contains $(A, E)$ and int$(A, E)$ is the largest fuzzy soft open set over X which is contained in $(A, E)$

Throughout the paper, the spaces X and Y (or $(X, \tau, E)$ and $(Y, \upsilon, K)$) stand for fuzzy soft topological spaces assumed unless otherwise stated.

## FUZZY SOFT $\alpha$-OPEN SETS

In this section we introduce fuzzy soft $\alpha$-open set, properties of fuzzy soft $\alpha$-open set and relation between them

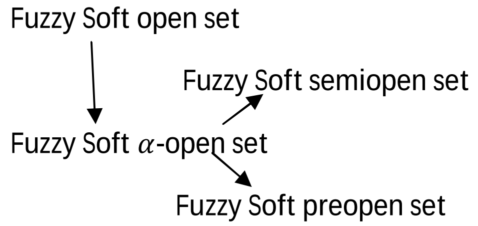
### Definition 9

A fuzzy soft set $(A, E)$ of a fuzzy soft topological space $(X, \tau, E)$ is called fuzzy soft $\alpha$-open set if $(A, E) \tilde{\sqsubseteq} int\Big(cl\big(int(A, E)\big)\Big)$. The complement of fuzzy soft $\alpha$-open set is called fuzzy soft $\alpha$-closed set.

### Definition 10

A fuzzy soft set $(A, E)$ is called fuzzy soft preopen set [9] (resp., fuzzy soft semi-open [5]) in a fuzzy soft topological space X if $(A, E) \tilde{\sqsubseteq} int\big(cl(A, E)\big)$ $\Big(\text{resp.}, (A, E) \tilde{\sqsubseteq} cl\big(int(A, E)\big)\Big)$.

We will denote the family of all fuzzy soft $\alpha$-open set $\left( \begin{matrix} resp., \alpha - \text{closed set and fuzzy soft} \\ \text{preopen sets} \end{matrix} \right)$ of a fuzzy soft topological space $(X, \tau, E)$ by $S_\alpha OS(X, \tau, E)$ $\big(resp., S_\alpha CS(X, \tau, E)$ and SPO$(X, \tau, E)\big)$.




### Proposition 11

(1) Arbitrary union of fuzzy soft $\alpha$-open sets is a fuzzy soft $\alpha$-open sets.

(2) Arbitrary intersection of fuzzy soft $\alpha$-closed sets is a fuzzy soft $\alpha$-closed sets.

Proof

(1) Let $\{(A_i, E): i \in \Lambda\}$ be a collection of fuzzy soft $\alpha$-open sets. Then, for each $i \in \Lambda$,

$(A_i, E) \mathrel{\tilde{\sqsubseteq}} int\left(cl\left(int(A_i, E)\right)\right)$. Now

$\tilde{\sqcup}\ (A_i, E) \mathrel{\tilde{\sqsubseteq}} \tilde{\sqcup}\ int\left(cl\left(int(A_i, E)\right)\right)$

$\mathrel{\tilde{\sqsubseteq}}\ int\ \tilde{\sqcup}\ cl\left(int(A_i, E)\right)$

$= int\ \left(cl\left(\tilde{\sqcup}\ int\ (A_i, E)\right)\right)$

$\mathrel{\tilde{\sqsubseteq}}\ int\left(cl\left(int\ \tilde{\sqcup}\ (A_i, E)\right)\right)$.

Hence $\tilde{\sqcup}\ (A_i, E)$ is a fuzzy soft $\alpha$-open set.

(2) Follows immediately from (1) by taking complements.

## Remark 12

It is obvious that every soft open $(resp., fuzzy\ soft\ closed)$ set is a fuzzy soft $\alpha$-open set$(resp., fuzzy\ soft\ \alpha -$ closed set). Similarly, every fuzzy soft $\alpha$-open set is fuzzy soft semiopen and fuzzy soft pre-open. Thus we have implications as shown in figure 1.

The examples given below show that the converses of these implications are not true.

## Example 13

Let $X = \{a, b, c\}, D = \{e_1, e_2, e_3\}$,

$A, B, C \sqsubseteq D$ where $A = \{e_1, e_2\}, B = \{e_2, e_3\}, C = \{e_1, e_3\}$ and Let $\mathfrak{T} = \{\tilde{1}_D, \tilde{0}_D, f_{1A}, f_{2B}, f_{3C}\}$, where $f_{1A}, f_{2B}, f_{3C}$ are fuzzy soft sets over $X$ defined as follows:

$\mu_{f_{1A}}^{e_1} = \{a_{0.2}, b_{0.8}, c_{0.5}\}, \mu_{f_{1A}}^{e_2} = \{a_{0.7}, b_0, c_{0.7}\}$,

$\mu_{f_{2B}}^{e_1} = \{a_{0.8}, b_{0.1}, c_1\}$,

$\mu_{f_{2B}}^{e_3} = \{a_{0.6}, b_{0.1}, c_{0.1}\}$,

$\mu_{f_{3C}}^{e_2} = \{a_{0.7}, b_{0.5}, c_{0.2}\}$.

Then, $\mathfrak{T}$ defines a fuzzy soft topology on $X$. Then the fuzzy soft set $k_D$ where

$\mu_{k_D}^{e_1} = \{a_{0.2}, b_{0.6}, c_{0.5}\},\ \mu_{k_D}^{e_2} = \{a_{0.7}, b_{0.1}, c_{0.9}\}$, $\mu_{k_D}^{e_1} = \{a_{0.6}, b_{0.5}, c_{0.1}\}$, is a fuzzy $\alpha$-open set of $(X, \mathfrak{T}, E)$, but is not a fuzzy soft open set.

## Definition 14

Let $(X, \tau, E)$ be a fuzzy soft topological space and let $(A, E)$ be a fuzzy soft set over X.

(1) Fuzzy soft $\alpha$-closure of a soft set $(A, E)$ in X is denoted by $S_\alpha cl\left((A, E)\right) =$ $\tilde{\sqcap}\ \{(F, E): (F, E)\ which\ is\ a\ fuzzy\ soft\ \alpha -$ closed set and $(A, E) \mathrel{\tilde{\sqsubseteq}} (F, E)\}$.

(2) Fuzzy soft $\alpha$-interior of a fuzzy soft set $(A, E)$ in X is denoted by $S_\alpha int\left((A, E)\right) =$ $\tilde{\sqcup}\ \{(O, E): (O, E)\ which\ is\ a\ fuzzy\ soft\ \alpha -$ open set and $(O, E) \mathrel{\tilde{\sqsubseteq}} (A, E)\}$.

Clearly $S_\alpha cl\left((A, E)\right)$ is the smallest fuzzy soft $\alpha$-closed set over X which contains $(A, E)$ and $S_\alpha int\left((A, E)\right)$ is the largest fuzzy soft $\alpha$-open set over X which is contained in $(A, E)$.

## Proposition 15

Let $(X, \tau, E)$ be a fuzzy soft topological space and let $(A, E)$ be a fuzzy soft set over X; then

(1) $(A, E) \in S_\alpha CS(X, \tau, E) \Leftrightarrow (A, E) = S_\alpha cl(A, E)$;

(2) $(A, E) \in S_\alpha OS(X, \tau, E) \Leftrightarrow (A, E) = S_\alpha int(A, E)$

Proof

(1) Let $(A, E) = S_\alpha cl\left((A, E)\right) = \tilde{\sqcap} \left\{ \begin{array}{c} (F, E): (F, E)\ be\ a\ fuzzy\ soft \\ \alpha - \text{closed set and } (A, E) \mathrel{\tilde{\sqsubseteq}} (F, E) \end{array} \right\}$.

This shows that

$(A, E) \in$

$\{(F, E): (F, E)\ is\ a\ fuzzy\ soft\ \alpha -$ closed set and $(A, E) \mathrel{\tilde{\sqsubseteq}} (F, E)\}$.

Hence $(A,E)$ is fuzzy soft $\alpha$-closed.

Conversely, let $(A,E)$ be fuzzy soft $\alpha$-closed set. Since $(A,E)\,\tilde{\sqsubseteq}\,(A,E)$ and $(A,E)$ is a fuzzy soft $\alpha$-closed,
$(A,E)\in\{(F,E):(F,E)\ is\ a\ fuzzy\ soft\ \alpha-$ closed set and $(A,E)\,\tilde{\sqsubseteq}\,(F,E)\}$.

Further, $(A,E)\,\tilde{\sqsubseteq}\,(F,E)$ for all such $(F,E)'$s.

$(A,E)=\tilde{\sqcap}$

$\{(F,E):(F,E)\ is\ a\ fuzzy\ soft\ \alpha-$ closed set and $(A,E)\,\tilde{\sqsubseteq}\,(F,E)\}$.

(2) similar to (1).

## Theorem 16

Let $(X,\tau,E)$ be a fuzzy soft topological space and let $(G,E)$ and $(K,E)$ be two fuzzy soft sets over X; then

(1) $S_\alpha cl\big((G,E)\big)^c = S_\alpha int\ (G,E)^c$;

(2) $S_\alpha int\big((G,E)\big)^c = S_\alpha cl\ (G,E)^c$;

(3) $(G,E)\,\tilde{\sqsubseteq}\,(K,A) \Rightarrow$

$S_\alpha int(G,E)\,\tilde{\sqsubseteq}\,S_\alpha int(K,E)$;

(4) $S_\alpha cl(\Phi) = \Phi\ and$

$S_\alpha cl(\tilde{X}) = \tilde{X}$;

(5) $S_\alpha int(\Phi) = \Phi\ and$

$S_\alpha int(\tilde{X}) = \tilde{X}$;

(6) $S_\alpha cl\big((G,E)\,\tilde{\sqcup}\,(K,E)\big) =$

$S_\alpha cl(G,E)\,\tilde{\sqcup}\,S_\alpha cl(K,E)$;

(7) $S_\alpha int\big((G,E)\,\tilde{\sqcap}\,(K,E)\big) =$

$S_\alpha int(G,E)\,\tilde{\sqcap}\,S_\alpha int(K,E)$;

(8) $S_\alpha cl\big((G,E)\,\tilde{\sqcap}\,(K,E)\big)\,\tilde{\sqsubseteq}$

$S_\alpha cl(G,E)\,\tilde{\sqcap}\,S_\alpha cl(K,E)$;

(9) $S_\alpha int\big((G,E)\,\tilde{\sqcup}\,(K,E)\big)\,\tilde{\sqsubseteq}$

$S_\alpha int(G,E)\,\tilde{\sqcup}\,S_\alpha int(K,E)$;

(10) $S_\alpha cl\Big(\big(S_\alpha cl(G,E)\big)\Big) = S_\alpha cl(G,E)$;

(11) $S_\alpha int\Big(\big(S_\alpha int(G,E)\big)\Big) =$

$S_\alpha int(G,E)$.

Proof

Let $(G,E)$ and $(K,E)$ be two fuzzy soft sets over X.

(1) $\big(S_\alpha cl(G,E)\big)^c =$

$(\tilde{\sqcap}\,\{(F,E)\mid(G,E)\,\tilde{\sqsubseteq}\,(F,E)\ and\ (F,E)\in S_\alpha CS(X,\tau,E)\})^c$

$=\tilde{\sqcup}\,\{(F,E)^c\mid(G,E)\,\tilde{\sqsubseteq}\,(F,E)\ and\ (F,E)\in S_\alpha CS(X,\tau,E)\} =$
$\tilde{\sqcup}\,\{(F,E)^c\mid(F,E)^c\,\tilde{\sqsubseteq}\,(G,E)^c\ and\ (F,E)^c\in S_\alpha OS(X,\tau,E)\} = S_\alpha int(G,E)^c$

(2) Similar to (1).

(3) It follows from Definition 15.

(4) Since $\Phi$ and $\tilde{X}$ are fuzzy soft $\alpha$-closed sets so, $S_\alpha cl(\Phi) = \Phi$ and $S_\alpha cl(\tilde{X}) = \tilde{X}$.

(5) Since $\Phi$ and $\tilde{X}$ are fuzzy soft $\alpha$-open sets so, $S_\alpha int(\Phi) = \Phi$ and $S_\alpha int(\tilde{X}) = \tilde{X}$.

(6) We have $(G,E)\,\tilde{\sqsubseteq}\,\big((G,E)\,\tilde{\sqcup}\,(K,E)\big)$ and $(K,E)\,\tilde{\sqsubseteq}\,\big((G,E)\,\tilde{\sqcup}\,(K,E)\big)$.

Then by proposition $S_\alpha cl(G,E)\,\tilde{\sqsubseteq}\,S_\alpha cl\big((G,E)\,\tilde{\sqcup}\,(K,E)\big)$ and $S_\alpha cl(K,E)\,\tilde{\sqsubseteq}\,S_\alpha cl\big((G,E)\,\tilde{\sqcup}\,(K,E)\big)\Rightarrow$
$S_\alpha cl(K,E)\,\tilde{\sqsubseteq}\,S_\alpha cl(G,E)\,\tilde{\sqsubseteq}\,S_\alpha$

$cl\big((A,E)\,\tilde{\sqcup}\,(K,E)\big)$.

Now, $S_\alpha cl(G,E), S_\alpha cl(K,E)\epsilon S_\alpha CS(X,\tau,E)\Rightarrow$ $S_\alpha cl(G,E)\,\tilde{\sqcup}\,S_\alpha cl(K,E)\in S_\alpha CS(X,\tau,E)$.

Then $(G,E)\,\tilde{\sqsubseteq}\,S_\alpha cl(G,E)$ and $(K,E)\,\tilde{\sqsubseteq}\,S_\alpha cl(K,E)$ imply $(G,E)\,\tilde{\sqcup}\,(K,E)\,\tilde{\sqsubseteq}\,S_\alpha cl(G,E)\,\tilde{\sqcup}\,S_\alpha cl(K,E)$. That

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is, $S_\alpha cl(G,E) \tilde{\sqcup} S_\alpha cl(K,E)$ is a fuzzy soft $\alpha$-closed set containing$(G,E) \tilde{\sqcup} (K,E)$.

But $S_\alpha cl\big((A,E) \tilde{\sqcup} (K,E)\big)$ is the smallest fuzzy soft $\alpha$-closed set containing$(G,E) \tilde{\sqcup} (K,E)$.

Hence $S_\alpha cl\big((G,E) \tilde{\sqcup} (K,E)\big) \tilde{\sqsubseteq} S_\alpha cl(G,E)$

$\tilde{\sqcup} S_\alpha cl(K,E)$. So, $S_\alpha cl\big((A,E)\big) S_\alpha cl\big((A,E) \tilde{\sqcup} (K,E)\big) = S_\alpha cl(G,E) \tilde{\sqcup} S_\alpha cl(K,E)$.

(7) Similar to (6)

(8) We have $\big((G,E) \tilde{\sqcap} (K,E)\big) \tilde{\sqsubseteq} (G,E)$ and $\big((G,E) \tilde{\sqcap} (K,E)\big) \tilde{\sqsubseteq} (K,E) \Rightarrow S_\alpha cl\big((G,E) \tilde{\sqcap} (K,E)\big)$

$\tilde{\sqsubseteq} S_\alpha cl(G,E)$ and $S_\alpha cl\big((G,E) \tilde{\sqcap} (K,E)\big)$

$\tilde{\sqsubseteq} S_\alpha cl(K,E). \Rightarrow S_\alpha cl\big((G,E) \tilde{\sqcap} (K,E)\big) \tilde{\sqsubseteq}$

$S_\alpha cl(G,E) \tilde{\sqsubseteq} S_\alpha cl(K,E)$.

(9) Similar to (8).

(10) Since $\big(S_\alpha cl(G,E)\big) \epsilon S_\alpha CS(X,\tau,E)$ so by proposition 16(1), $S_\alpha cl\Big(\big(S_\alpha cl(G,E)\big)\Big) = \big(S_\alpha cl(G,E)\big)$.

(11) Since $\big(S_\alpha int(G,E)\big) \epsilon S_\alpha OS(X,\tau,E)$ so by proposition 16(2), $S_\alpha int\Big(\big(S_\alpha int(G,E)\big)\Big) = \big(S_\alpha int(G,E)\big)$.

## Theorem 17

If $(G,E)$ is any fuzzy soft set in a fuzzy soft topological space $(X,\tau,E)$, then following are equivalent:

(1) $(G,E)$ is fuzzy soft $\alpha$-closed set;

(2) $int\big(cl(int(G,E)^c)\big) \tilde{\sqsupseteq} (G,E)^c$;

(3) $cl\Big(int\big(cl(G,E)\big)\Big) \tilde{\sqsubseteq} (G,E)$;

(4) $(G,E)$ is fuzzy soft $\alpha$-open set.

Proof

(1) $\Rightarrow$ (2) If $(G,E)$ is fuzzy soft $\alpha$-closed set, then $cl\Big(int\big(cl(G,E)\big) \tilde{\sqsubseteq} (G,E)\Big) \Rightarrow$
$(G,E)^c \tilde{\sqsubseteq} int\big(cl(int(G,E)^c)\big)$;

(2) $\Longrightarrow$ (3)
$\Big(int\big(cl(int(G,E)^c)\big)^c \tilde{\sqsupseteq} ((G,E)^c)\Big)^c \Rightarrow$
$cl\Big(int\big(cl(G,E)\big)\Big) \tilde{\sqsubseteq} (G,E)$;

(3) $\Longrightarrow$ (4) It is obvious from Definition 9;

(4) $\Longrightarrow$ (1) It is obvious from Definition 9.

# FUZZY SOFT $\alpha$-CONTINUITY

In this section we introduce the fuzzy soft $\alpha$-continuous, properties & relations

## Definition 18

Let $(X,E)$ and $(Y,K)$ be fuzzy soft classes. Let $u : X \to Y$ and $p : E \to K$ be mapping $f : (X,E) \to (Y,K)$ is defined as follows: for a fuzzy soft set $(F,A)$ in $(X,E)$, $(f(F,A),B), B = p(A) \sqsubseteq K$ is a fuzzy soft set in $(Y,K)$ given by $f(F,A)(\beta) = u\left(\underset{\alpha\epsilon p^{-1}(\beta)\cap A}{UF(\alpha)}\right)$ for $\beta\epsilon K$. $(f(F,A),B), B$ is called fuzzy soft image of a soft set $(F,A)$. If $B = K$, then we will write $(f(F,A),K)$ as $f(F,A)$.

## Definition 19

Let $f : (X,E) \to (Y,K)$ be a mapping from a fuzzy soft class $(X,E)$ to another fuzzy soft class $(Y,K)$ and $(G,C)$ of a fuzzy soft set in fuzzy soft class$(Y,K)$, where $C \sqsubseteq K$. Let $u : X \to Y$ and $p : E \to K$ be mappings. Then $(f^{-1}(G,C),D), D = p^{-1}(C)$, is a fuzzy soft set in fuzzy soft classes $(X,E)$, defined as $f^{-1}(G,C)(\alpha) = u^{-1}\Big(G\big(p(\alpha)\big)\Big)$ for $\alpha \epsilon D \sqsubseteq E$. $(f^{-1}(G,C),D)$ is called a fuzzy soft inverse image of $(G,C)$. Hereafter we will write $(f^{-1}(G,C),E)$ as $f^{-1}(G,C)$.

## Theorem 20

Let $f : (X, E) \to (Y, K), u : X \to Y$ and $p : E \to K$ be mappings. Then for fuzzy soft sets $(F, A)$ and $(G, B)$ and a family of fuzzy soft sets $(F_i, A_i)$ in the fuzzy soft class $(X, E)$, one has:

(1) $f(\Phi) = \Phi$,

(2) $f(\tilde{X}) = \tilde{Y}$,

(3) $f((F, A) \tilde{\sqcup} (G, B)) = f(F, A) \tilde{\sqcup} f(G, B)$ in general $f(\sqcup_i (F_i, A_i)) = \sqcup_i f(F_i, A_i)$,

(4) $f((F, A) \tilde{\sqcup} (G, B)) \tilde{\sqsupseteq} f(F, A) \tilde{\sqcap} f(G, B)$ in general $f(\sqcap_i (F_i, A_i)) \tilde{\sqsubseteq} \sqcap_i f(F_i, A_i)$

(5) If $(F, A) \tilde{\sqsubseteq} (G, B)$, then $f(F, A) \tilde{\sqsubseteq} f(G, B)$,

(6) $f^{-1}(\Phi) = \Phi$,

(7) $f^{-1}(Y) = \tilde{X}$,

(8) $f^{-1}((F, A) \tilde{\sqcup} (G, B)) = f^{-1}(F, A) \tilde{\sqcup} f^{-1}(G, B)$ in general $f^{-1}(\sqcup_i (F_i, A_i)) = \sqcup_i f^{-1}(F_i, A_i)$,

(9) $f^{-1}((F, A) \tilde{\sqcap} (G, B)) = f^{-1}(F, A) \tilde{\sqcup} f^{-1}(G, B)$ in general $f^{-1}(\sqcap_i (F_i, A_i)) = \sqcap_i f^{-1}(F_i, A_i)$,

(10) If $(F, A) \tilde{\sqsubseteq} (G, B)$, then $f^{-1}(F, A) \tilde{\sqsubseteq} f^{-1}(G, B)$.

## Definition 21

A mapping $f : (X, \tau, E) \to (Y, \upsilon, K)$ is said to be fuzzy soft mapping if $(X, \tau, E)$ and $(Y, \upsilon, K)$ are fuzzy soft topological spaces and $u : X \to Y$ and $p : E \to K$ are mappings.

Throughout the paper, the spaces X and Y $(or(X, \tau, E)\ and\ (Y, \upsilon, K))$ stand for fuzzy soft topological space assumed unless otherwise stated.

## Definition 22

A soft mapping $f : X \to Y$ is said to be fuzzy soft $\alpha$-continuous if the inverse Image of each fuzzy soft $\alpha$-open subset of Y is a fuzzy soft $\alpha$-open set in X.

## Example 23

Let $X = \{x_1, x_2, x_3\}, Y = \{y_1, y_2, y_3\}, E = \{e_1, e, e_3\}, K = \{k_1, k_2, k_3\}, \tau = \{\Phi, \tilde{X}, (F, E)\}, (F, E) = \{(e_1, \{x_1\}), (e_2, \{x_2\}), (e_3, \{x_1, x_3\})\}, \upsilon = \{\Phi, \tilde{Y}, (G, K)\}$, and $(G, K) = \{(k_1, \{y_1, y_3\}), (k_2, \{y_1\}), (k_3, \{y_3\})\}$ and let $(X, \tau, E)$ and $(Y, \upsilon, K)$ be fuzzy soft topological spaces.

Define $u : X \to Y$ and $p : E \to K$ as

$u(x_1) = \{y_1\}, u(x_2) = \{y_3\}, u(x_3) = \{y_2\}$,

$p(e_1) = \{y_1\}, p(e_2) = \{k_3\}, p(e_3) = \{k_1\}$

Let $f_{up} : (X, \tau, E) \to (Y, \upsilon, K)$ be a soft mapping. Then $(G, K)$ is a fuzzy soft open in Y and $f_{pu}^{-1}((G, K)) = (F, E)$ is a fuzzy soft $\alpha$-open in X. Therefore, $f_{pu}$ is a fuzzy soft $\alpha$-continuous function.

## Theorem 24

Let $f : X \to Y$ be a mapping from a fuzzy soft space X to soft space Y. Then the following statements are true:

(1) f is fuzzy soft $\alpha$-continuous;

(2) for each fuzzy soft singleton $(P, E)$ in X and each fuzzy soft open set $(O, K)$ in Y and $f((P, E)) \tilde{\sqsubseteq} (O, K)$, there exists a fuzzy soft $\alpha$-open set $(U, E)$ in X such that $(P, E) \tilde{\sqsubseteq} (U, E)$ and $f((U, E)) \tilde{\sqsubseteq} (O, K)$;

(3) The inverse image of each fuzzy soft closed set in Y is fuzzy soft $\alpha$-closed in X;

(4) $f\left(cl\left(int\left(cl(A, E)\right)\right)\right) \tilde{\sqsubseteq} cl(f(A, E))$, for each fuzzy soft set $(A, E)$ in X;

(5) $cl\left(int\left(cl(f^{-1}(B, K))\right)\right) \tilde{\sqsubseteq} f^{-1}(cl(B, K))$, for each fuzzy soft set $(B, K)$ in Y.

Proof

(1) $\Rightarrow$ (2) Since $(O,K)$ is fuzzy soft open in Y and $f((P,E)) \sqsubseteq (O,K)$, so $(P,E) \sqsubseteq f^{-1}((O,K))$ and $f^{-1}((O,K))$ is a fuzzy soft $\alpha$-open set in X. Put $(U,E) = f^{-1}((O,K))$. Then $(P,E) \sqsubseteq (U,E)$ and $f((U,E)) \sqsubseteq (O,K)$.

(2) $\Longrightarrow$ (1) Let $(O,K)$ be a fuzzy soft open set in Y such that $(P,E) \sqsubseteq f^{-1}((O,K))$ and thus there exists $(U,E) \epsilon\ SaOS(X)$ such that $(P,E) \sqsubseteq (U,E)$ and $f((U,E)) \sqsubseteq (O,K)$ Then $(P,E) \sqsubseteq (U,E) \sqsubseteq f^{-1}((O,K)) = \tilde{\sqcup}\,(U,E) \in S_\alpha(X)$. Hence $f^{-1}((O,K)) \in SaOS(X)$ and therefore $f$ is fuzzy soft $\alpha$-continuous.

(1) $\Rightarrow$ (3) Let $(G,K)$ be a fuzzy soft closed set in Y. Then $(G,K)^c$ is fuzzy soft open in Y. Thus $f^{-1}((G,K)^c) \epsilon\ SaOS(X)$. Hence $f^{-1}((G,K))$ is a fuzzy soft $\alpha$-set in X.

(3) $\Rightarrow$ (4) Let $(S,K)$ be a fuzzy soft set in Y. Then $cl(f(S,K))$ is a fuzzy soft closed set in Y, so that $f^{-1}(cl(S,K))$ is fuzzy soft $\alpha$-closed in X.

Therefore, we have $f^{-1}(cl(S,K)) \tilde{\sqsupseteq}$

$$cl\left(int\left(cl\left(f^{-1}(cl(S,K))\right)\right)\right) \tilde{\sqsupseteq}$$

$$cl\left(int\left(cl(cl(S,K))\right)\right) = cl\left(int(cl(S,K))\right).$$

(4) $\Rightarrow$ (5) Since $(B,K)$ be a fuzzy soft set in Y, then $f^{-1}((B,K))$ is a fuzzy soft set in X; thus by hypothesis we have $cl\left(int\left(cl\left(f\left(f^{-1}((B,K))\right)\right)\right)\right) \sqsubseteq$

$cl\left(f\left(f^{-1}((B,K))\right)\right)$ or $cl\left(int\left(cl\left(f\left(f^{-1}((B,K))\right)\right)\right)\right)$

$\sqsubseteq cl(B,K)$ that is, $cl\left(int\left(cl\left(f^{-1}((B,K))\right)\right)\right) \sqsubseteq f^{-1}(cl(B,K))$.

(5) $\Rightarrow$ (1) Let $(O,K)$ be a soft open in Y. Let $(U,K) = (O,K)^c$ and $(D,E) = f^{-1}((U,K))$. By (5) we have $cl\left(int\left(cl\left(f\left(f^{-1}((U,K))\right)\right)\right)\right)$

$\tilde{\sqsubseteq}\ cl(U,K) = (U,K)$; that is; $cl\left(int\left(cl(f^{-1}((O,K)^c))\right)\right) \sqsubseteq$

$f^{-1}((O,K)^c)$. Therefore $f^{-1}((O,K))$ is a fuzzy soft $\alpha$-open set in X; hence $f$ is fuzzy soft $\alpha$-continuous function.

## Corollary 25

Let $f: X \to Y$ be a fuzzy soft $\alpha$-continuous mapping. Then

(1) $f(cl(A,E)) \sqsubseteq cl\left(f(cl(A,E))\right)$, for each $(A,E) \in SPO(X)$;

(2) $cl\left(f^{-1}((B,K))\right) \sqsubseteq f^{-1}(cl(B,K))$, for each $(B,K) \in SPO(Y)$.

Proof

Since for each $(A,E) \in SPO(X), cl((A,E)) = cl\left(int\left(cl((A,E))\right)\right)$, therefore the proof follows directly from statements (4) and (5) of theorem 26.

## Definition 26

A Fuzzy Soft mapping $f: X \to Y$ is called fuzzy soft pre-continuous (resp., fuzzy soft semicontinuous [7]) if the inverse image of each fuzzy soft open set in Y is fuzzy soft preopen (resp., fuzzy soft semiopen) in X.

## Remark 27

It is clear that every fuzzy soft $\alpha$-continuous map is fuzzy soft semicontinuous and fuzzy soft precontinuous. Every fuzzy soft continuous map is fuzzy soft $\alpha$-continuous. Thus we have implications as shown in fig 2.

The converses of these implications are not true, which is clear from the following examples.

## Example 28

Let $X = \{x_1, x_2, x_3, x_4\}, Y = \{y_1, y_2, y_3, y_4\}, E = \{e_1, e_2, e_3\}, K = \{k_1, k_2, k_3\}$ and $(X, \tau, E)$ and Let $(Y, \upsilon, K)$ be fuzzy soft topological spaces.

$$\begin{array}{ccc} \text{Fuzzy Soft continuity} & & \\ \downarrow & \nearrow & \text{Fuzzy Soft semicontinuity} \\ \text{Fuzzy Soft } \alpha\text{-continuity} & & \\ & \searrow & \text{Fuzzy Soft precontinuity} \end{array}$$

Define $u : X \to Y$ and $p : E \to K$ as

$u(x_1) = \{y_2\}, u(x_2) = \{y_4\}, u(x_3) = \{y_1\}, u(x_4) = \{y_3\},$

$p(e_1) = \{k_2\}, p(e_2) = \{k_1\}, p(e_3) = \{k_3\}.$

Let us consider the fuzzy soft topology $\tau$ given in example 14; that is, $\tau = \{\Phi, \tilde{X}, (F_1, E), (F_2, E), (F_3, E), \ldots, (F_{15}, E)\}$,

$\upsilon = \{\Phi, \tilde{Y}, (F, K)\}\ and\ (F, K) = \{(k_1, \{y_1, y_3, y_4\}), (k_2, \{y_1, y_2, y_4\}), (k_3, \{y_2, y_4\})\}$ and let mapping $f_{up} : (X, \tau, E) \to (Y, \upsilon, K)$ be a fuzzy soft mapping. Then $(F, K)$ is a fuzzy soft open in Y and $f_{pu}^{-1}((F, K)) = \{(e_1, \{x_1, x_2, x_3\}), (e_2, \{x_2, x_3, x_4\}), (e_3, \{x_1, x_2, \})\}$ is a fuzzy soft $\alpha$-open but not fuzzy soft open in X. Therefore, $f_{pu}$ is a fuzzy soft $\alpha$-continuous function but not fuzzy soft continuous function.

Therefore, $f_{pu}$ is a fuzzy soft $\alpha$-continuous function but not fuzzy soft continuous function.

## Example 29

Let $X = \{a, b, c\}, Y = \{x, y\}, E = \{e_1, e_2\}$ and $K = \{k_1, k_2\}$. Define

$A \sqsubseteq E, B \sqsubseteq K, f_A = \{(e_1, \{0.5, 0.5, 0.3\}), (c_2, \{0.4, 0.4, 0.2\})\}$

$\in (\widetilde{X, E})$ and

$g_B = \{(k_1\{0.5, 0.3\}), (k_2, \{0.4, 0.2\})\} \in (\widetilde{Y, K})$

Define fuzzy soft topologies $\tau_E : E \to \mathfrak{T}(\widetilde{X, E})$ and $\tau_k^* : K \to \mathfrak{T}(\widetilde{Y, K})$ as follows:

$$\tau_e(h_G) = \begin{cases} 1, if\ h_G = \varphi, \tilde{E} \\ \frac{1}{2}\ if\ h_G = f_{A,} \widetilde{E^{0.4}} \\ \frac{1}{2}\ if\ h_G = f_A \sqcap \widetilde{E^{0.4}} \\ \frac{2}{3}\ if\ h_G = f_A \sqcup \widetilde{E^{0.4}} \\ 0, Otherwise \end{cases}$$

$$\tau_k^*(w_D) = \begin{cases} 1, if\ w_D = \varphi, \tilde{E} \\ \frac{1}{2}, if\ w_D = g_B \\ 0, Otherwise \end{cases}$$

Consider the maps $\varphi: X \to Y$ and $\Psi: E \to K$ defined by $\varphi(a) = \varphi(b) = X, \varphi(c) = y, \Psi(e_1) = k_1$ and $\Psi(e_2) = k_2$

Therefore, for each $e \in E, k \in K$ and $r \in \mathfrak{T}_0$ define the fuzzy soft operators $\alpha = E \times (\widetilde{X, E}) \times \mathfrak{T}_0 \to (\widetilde{X, E})$ as follows:

$\alpha(e, w_D, r) = w_D,$

Then the map $\varphi_\Psi: (X, e_E) \to (Y, \tau_k^*)$ is fuzzy soft continuous.

## Example 30

Let $X = Y = \{a, b, c\}, E = \{e_1, e_2, e_3\}$ and $A \sqsubseteq E$ where $A = \{e_1, e_2\}$. Let $f_{pu}: (X, \mathfrak{T}_1, E) \to (Y, \mathfrak{T}_2, K)$ be the constant soft mapping where $\mathfrak{T}_1$ is the indiscrete fuzzy soft topology and $\mathfrak{T}_2$ is the discrete fuzzy soft topology such that $u(x) = a\ \forall x \in X$ and $p(e) = e_1 \forall e \in E$. Let $f_A$ be fuzzy soft set over $Y$ defined as follows:

$\mu_{f_A}^{e_1} = \{a_{0.1}, b_{0.5}, c_{0.6}\}$

$\mu_{f_A}^{e_2} = \{a_{0.6}, b_{0.2}, c_{0.5}\}$

Then $f_A \in \mathfrak{T}_2$. Now we find $f_{pu}^{-1}(f_A)$ as follows

$f_{pu}^{-1}(f_A)(e_1)(a) = f_A(p(e_1))(u(a))$

$= f_A(e_1)(a)$

$= 0.6$

$f_{pu}^{-1}(f_A)(e_1)(b) = f_A(p(e_1))(u(b))$

$= f_A(e_1)(a)$

$= 0.6$

$f_{pu}^{-1}(f_A)(e_1)(c) = f_A(p(e_1))(u(c))$

$= f_A(e_1)(a)$

$= 0.6$

$f_{pu}^{-1}(f_A)(e_2)(a) = f_A(p(e_2))(u(a))$

$= f_A(e_1)(a)$

$= 0.6$

$f_{pu}^{-1}(f_A)(e_2)(b) = f_A(p(e_2))(u(b))$

$= f_A(e_1)(a)$

$= 0.6$

$f_{pu}^{-1}(f_A)(e_2)(c) = f_A(p(e_2))(u(c))$

$= f_A(e_1)(a)$

$= 0.6$

$f_{pu}^{-1}(f_A)(e_3)(a) = f_A(p(e_3))(u(a))$

$= f_A(e_1)(a)$

$= 0.6$

$f_{pu}^{-1}(f_A)(e_3)(b) = f_A(p(e_3))(u(b))$

$= f_A(e_1)(a)$

$= 0.6$

$f_{pu}^{-1}(f_A)(e_3)(c) = f_A(p(e_3))(u(c))$

$= f_A(e_1)(a)$

$= 0.6$

Hence $f_{pu}^{-1}(f_A) \notin F\alpha OS(X)$

Therefore $f_{pu}$ is not fuzzy soft $\alpha$-continuous function. On the other hand, if we consider $\mathfrak{T}_1$ is the discrete fuzzy soft topology. In this case $f_{pu}$ will be fuzzy $\alpha$-continuous soft function

## Theorem 31

Let $(A,E) \in SPO(X)$ and $(B,E) \epsilon S\alpha OS(X)$. Then $(A,E) \tilde{\sqcap} (B,E) \in S\alpha OS(X)$.

Proof

Since
$(A,E) \tilde{\sqcap} (B,E) \tilde{\sqsubseteq} int(cl(A,E)) \tilde{\sqcap} int(cl(B,E)) = int\left(int(cl(A,E))\right) \tilde{\sqcap} cl(int(B,E)) \tilde{\sqsubseteq}$

$int\left(cl\left((A,E) \tilde{\sqcap} int(cl(B,E))\right)\right)$, it follows that $(A,E) \tilde{\sqcap} (B,E) \tilde{\sqsubseteq} int\left(cl((A,E) \tilde{\sqcap} int(B,E))\right)$

$\tilde{\sqcap} (A,E)$.

$$= s\alpha\ int\begin{pmatrix} int\left(cl((A,E) \tilde{\sqcap} int(B,E))\right) \\ \tilde{\sqcap} (A,E) \end{pmatrix}$$

$$\tilde{\sqsubseteq} s\alpha int\left(cl((A,E) \tilde{\sqcap} int(B,E)) \tilde{\sqcap} (A,E)\right)$$

$$= s\alpha\ int\left(s\alpha\ cl((A,E) \tilde{\sqcap}\ int(B,E))\right)$$

$$= s\alpha\ int\begin{pmatrix} s\alpha\ cl \\ \left(s\alpha\ int((A,E) \tilde{\sqcap} int(B,E))\right) \end{pmatrix}$$

$$\tilde{\sqsubseteq} s\alpha\ int\left(s\alpha\ cl\left(s\alpha\ int\begin{pmatrix} (A,E) \\ \tilde{\sqcap} (B,E) \end{pmatrix}\right)\right).$$

Therefore $(A,E) \tilde{\sqcap} (B,E)$ is a fuzzy soft $\alpha$-open set of $(A,E)$.

## Theorem 32

If $f : X \to Y$ is a fuzzy soft $\alpha$-continuous mapping and $(A,E) \in SPO(X)$, then $f|_{(A,E)}$ is fuzzy soft $\alpha$-continuous mapping.

Proof

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Let $(B, E)$ in Y be a fuzzy soft open set. Then $f^{-1}((B, E)) \in S_\alpha(X)$ and since $(A, E)$ is a fuzzy soft preopen set in X, by theorem 26, we have $(A, E) \tilde{\sqcap} f^{-1}((B, E)) = (f|_{(A,E)})^{-1}((B, E)) \in S\alpha OS(X)((A, E))$. Therefore $f|_{(A,E)}$ is a fuzzy soft $\alpha$-continuous mapping.

## Theorem 33

A fuzzy soft function $f : X \to Y$ is fuzzy soft $\alpha$-continuous if and only if $f(s\alpha\, cl(F, E)) \tilde{\sqsubseteq}\ cl\ (f((F, E)))$ for every fuzzy soft set $(F, E)$ of X.

Proof

Let $f : X \to Y$ be fuzzy soft $\alpha$-continuous. Now $cl\left(f((F, E))\right)$ is a fuzzy soft closed set of Y;

So by fuzzy soft $\alpha$-continuity of $f$, $f^{-1}\left(cl\left(f((F, E))\right)\right)$ is fuzzy soft $\alpha$-closed and $(F, E) \tilde{\sqsubseteq}\ f^{-1}\left(cl\left(f((F, E))\right)\right)$.

But $s\alpha\, cl(F, E)$ is the smallest $\alpha$-closed set containing $(F, E)$;

Hence $s\alpha\ cl\ (F, E) \tilde{\sqsubseteq} f^{-1}\left(cl\left(f((F, E))\right)\right)$

$\Rightarrow f(s\alpha\, cl(F, E)) \tilde{\sqsubseteq}\ cl\left(f((F, E))\right)$.

Conversely, Let $(F, K)$ be any fuzzy soft closed set of Y

$\Rightarrow\ f^{-1}((F, K))\ \in X$ and by hypothesis

$\Rightarrow f\left(s\alpha\, cl\left(f^{-1}((F, K))\right)\right) \tilde{\sqsubseteq}$

$cl\left(f\left(f^{-1}((F, K))\right)\right)$

$\Rightarrow f\left(s\alpha\, cl\left(f^{-1}((F, K))\right)\right) \tilde{\sqsubseteq}$

$cl(F, K) = (F, K)$

$\Rightarrow s\alpha\ cl\left(f^{-1}((F, K))\right) = f^{-1}((F, K))$; hence fuzzy soft $\alpha$-closed. Consequently, $f$ fuzzy soft $\alpha$-continuous.

## Theorem 34

A Fuzzy Soft function $f : X \to Y$ is fuzzy soft $\alpha$-continuous if and if $f^{-1}(int(H, K)) \tilde{\sqsubseteq}\ s\alpha\ int f^{-1}((H, K))$ for every fuzzy soft set $(H, K)$ of Y.

Proof

Let $f : X \to Y$ be fuzzy soft $\alpha$-continuous. Now for any fuzzy soft set $(G, E)$ in X, $int\left(f((G, E))\right)$ is a fuzzy soft open set in Y; since $f$ is fuzzy soft $\alpha$-continuity, then $f^{-1}\left(int(f(G, E))\right)$ is fuzzy soft $\alpha$-open and $f^{-1}\left(int(f(G, E))\right) \tilde{\sqsubseteq} (G, E)$, As $s\alpha\ int(G, E)$ is the largest fuzzy soft $\alpha$-open set contained in $(G, E)$, $f^{-1}\left(int(f(G, E))\right) \tilde{\sqsubseteq}\ s\alpha\ int(G, E)$.

Conversely, take a fuzzy soft open set $(G, K)$ in Y. Then $f^{-1}(int(G, K)) \tilde{\sqsubseteq}\ s\alpha\ int f^{-1}(G, K) \Rightarrow f^{-1}(G, K) \tilde{\sqsubseteq} s\alpha\ int f^{-1}(G, K) \Rightarrow f^{-1}(G, K)$ is fuzzy soft $\alpha$-open.

## FUZZY SOFT $\alpha$-OPEN AND FUZZY SOFT $\alpha$-CLOSED MAPPINGS

## Definition 35

A fuzzy soft mapping $f : X \to Y$ is called fuzzy soft $\alpha$-open (resp., fuzzy soft $\alpha$-closed) mapping if the image of each fuzzy soft open (resp., fuzzy soft closed) set in X is a fuzzy soft $\alpha$-open set (resp., fuzzy soft $\alpha$-closed set) in Y.

## Definition 36

A fuzzy soft mapping $f : X \to Y$ is called fuzzy soft pre-open (resp., fuzzy soft semiopen[7]) if the image of each fuzzy soft open set in X is fuzzy soft preopen (rep., fuzzy soft semiopen) in Y.

Clearly a fuzzy soft open map is fuzzy soft $\alpha$-open map is fuzzy soft preopen as well as fuzzy soft $\alpha$-open. Similar implications hold for fuzzy soft closed mappings.

### Theorem 37

A fuzzy soft mapping $f : X \to Y$ is fuzzy soft $\alpha$-closed if and only if $s\alpha cl\big(f(A,E)\big) \tilde{\sqsubseteq} f\big(cl(A,E)\big)$ for each fuzzy soft set $(A,E)$ in X.

Proof

Let $s\alpha cl\big(f(A,E)\big) \tilde{\sqsubseteq} f\big(cl(A,E)\big)$. By the definition of fuzzy soft $\alpha$-closure, we have $f\big((A,E)\big) = f\big(cl(A,E)\big)$ and so $f\big(cl(A,E)\big)$ is fuzzy soft $\alpha$-closed set and $f$ is a fuzzy soft $\alpha$-closed mapping.

Conversely, if $f$ is fuzzy soft $\alpha$-closed, then $f\big(cl(A,E)\big)$ is a fuzzy soft $\alpha$-closed set containing $f\big((A,E)\big)$ and therefore $s\alpha cl\ \big(f(A,E)\big) \tilde{\sqsubseteq} f\big(cl(A,E)\big)$.

### Theorem 38

A fuzzy soft function $f : X \to Y$ is fuzzy soft $\alpha$-open if and only if $f\big(int(F,E)\big) \tilde{\sqsubseteq}\ s\alpha\ int\left(f\big((F,E)\big)\right)$ for every fuzzy soft set $(F,A)$ in X.

Proof

If $f : X \to Y$ is fuzzy soft $\alpha$-open, then $f\big(int(F,E)\big) =$
$s\alpha\ int\ f\big(int(F,E)\big) \tilde{\sqsubseteq} s\alpha\ int\ f\big(int(F,E)\big)$

$\tilde{\sqsubseteq}\ s\alpha\ int\ f(F,E)$.

On the other hand, take a fuzzy soft open set $(G,E)$ in X. Then by hypothesis, $f\big((G,E)\big) = f\big(int(G,E)\big) \tilde{\sqsubseteq}\ s\alpha\ int\ \left(f\big((G,E)\big)\right) \Rightarrow$ $f\big((G,E)\big)$ is fuzzy soft $\alpha$-open in Y.

### Theorem 39

Let $f : X \to Y$ be a fuzzy soft $\alpha$-open (resp., fuzzy soft $\alpha$-closed) mapping. If $(B,K)$ is a fuzzy soft set in Y and $(A,E)$ is a fuzzy soft closed (resp., fuzzy soft open) set in X, containing $f^{-1}\big((B,K)\big)$; then there exists a fuzzy soft $\alpha$-closed (resp., fuzzy soft $\alpha$-open) set $(C,K)$ in Y, such that $(B,K) \tilde{\sqsubseteq}\ (C,K)$ and $f^{-1}\big((C,K)\big) \tilde{\sqsubseteq}\ (A,E)$.

Proof

Let $(C,K) = (f(A,E)^c)^c$. Since $f^{-1}\big((B,K)\big) \tilde{\sqsubseteq} (A,E)$, we have $f((A,E)^c) \tilde{\sqsubseteq} (B,K)^c$. Since $f$ is fuzzy soft $\alpha$-open (resp.,fuzzy soft $\alpha$-closed), then $(C,K)$ is a fuzzy soft $\alpha$-closed set (resp.,fuzzy soft $\alpha$-open set) if $f^{-1}\big((C,K)\big) =$
$\left(f^{-1}\big(f((A,E)^c)\big)\right)^c \tilde{\sqsubseteq} ((A,E)^c)^c = (A,E)$ and hence $(B,K) \tilde{\sqsubseteq} (C,K)$ and $f^{-1}\big((C,K)\big) \tilde{\sqsubseteq} (A,E)$.

### Corollary 40

If $f : X \to Y$ is a fuzzy soft $\alpha$-open mapping, then

(1) $f^{-1}\left(cl\left(int\left(cl\big((B,K)\big)\right)\right)\right) \tilde{\sqsubseteq}$

$cl\left(f^{-1}\big((B,K)\big)\right)$, for every fuzzy soft set $(B,K)$ in Y;

(2)
$f^{-1}\left(cl\big((C,K)\big)\right) \tilde{\sqsubseteq}\ cl\ \left(f^{-1}\big((C,K)\big)\right), (C,K) \in SPO(Y)$.

Proof

(1) $cl\left(f^{-1}\big((B,K)\big)\right)$ is a fuzzy soft closed in X, containing $f^{-1}\big((B,K)\big)$, for a fuzzy soft set $(B,E)$ in Y.

By theorem 44, there exists a fuzzy soft $\alpha$-closed set $(F,K)$ in Y, and $(B,K) \tilde{\sqsubseteq}\ (F,K)$ such that $f^{-1}\big((F,K)\big) \tilde{\sqsubseteq} cl\left(f^{-1}\big((B,K)\big)\right)$.

Thus $f^{-1}\left(cl\left(int\left(cl\big((B,E)\big)\right)\right)\right) \tilde{\sqsubseteq}$

$$cl\left(f^{-1}\left(cl\left(int\left(cl\big((F,E)\big)\right)\right)\right)\right) \tilde{\subseteq}$$

$$f^{-1}\big((F,K)\big) \tilde{\subseteq} cl\left(f^{-1}\big((B,K)\big)\right)$$

(2) follows easily from (1).

## Theorem 41

If $f : X \to Y$ is a fuzzy soft precontinuous and fuzzy soft $\alpha$-open mapping , then $f^{-1}\big((B,K)\big) \in SPO(X)$ for each $(B,K) \in SPO(Y)$.

Proof

We have $f^{-1}\big((B,K)\big) \tilde{\subseteq} f^{-1}\left(int\left(cl\big((B,K)\big)\right)\right)$

$$\tilde{\subseteq} int\left(cl\left(f^{-1}\left(int\left(cl\big((B,K)\big)\right)\right)\right)\right)$$

$$\tilde{\subseteq} int\left(cl\left(f^{-1}\left(cl\big((B,K)\big)\right)\right)\right)$$

Since $f$ is a soft $\alpha$-open map, we have, by corollary 42,

$$f^{-1}\big((B,K)\big) \tilde{\subseteq} int\left(cl\left(f^{-1}\left(cl\big((B,K)\big)\right)\right)\right)$$

$$\tilde{\subseteq} cl\left(int\left(f^{-1}\left(cl\big((B,K)\big)\right)\right)\right) =$$

$$cl\left(int\left(f^{-1}\big((B,K)\big)\right)\right).$$

Therefore $f^{-1}\big((B,K)\big)$ is a fuzzy soft preopen set in X.

## Theorem 42

If $f : X \to Y$ is a fuzzy soft precontinuous and fuzzy soft semicontinuous, then $f$ is fuzzy soft $\alpha$-continuous.

Proof

We have $f^{-1}\big((B,K)\big) \tilde{\subseteq} f^{-1}\left(int\left(cl\big((B,K)\big)\right)\right)$

$$\tilde{\subseteq} int\left(cl\left(f^{-1}\left(int\left(cl\big((B,K)\big)\right)\right)\right)\right)$$

$$\tilde{\subseteq} int\left(cl\left(f^{-1}\left(cl\big((B,K)\big)\right)\right)\right)$$

Since $f$ is a soft $\alpha$-open map, we have, by corollary 42,

$$f^{-1}\big((B,K)\big) \tilde{\subseteq} int\left(cl\left(f^{-1}\left(cl\big((B,K)\big)\right)\right)\right)$$

$$\tilde{\subseteq} cl\left(int\left(f^{-1}\left(cl\big((B,K)\big)\right)\right)\right) =$$

$$cl\left(int\left(f^{-1}\big((B,K)\big)\right)\right).$$

Therefore $f^{-1}\big((B,K)\big)$ is a fuzzy soft preopen set in X.

## Theorem 43

If $f : X \to Y$ is a fuzzy soft precontinuous and fuzzy soft semicontinuous, then $f$ is fuzzy soft $\alpha$-continuous.

Proof

Let $(B,K)$ be any fuzzy soft open set in Y. Then $f^{-1}\big((B,K)\big)$ is a fuzzy soft preopen set as well as a fuzzy soft semiopen set in X.

We have $f^{-1}\big((B,K)\big) \tilde{\subseteq} cl\left(int\left(f^{-1}\big((B,K)\big)\right)\right)$

and

$$f^{-1}\big((B,K)\big) \tilde{\subseteq} int\left(cl\left(int\left(cl\big(f^{-1}(B,K)\big)\right)\right)\right) =$$

$$int\left(cl\left(int\big(f^{-1}(B,K)\big)\right)\right).$$

Hence $f$ is a fuzzy soft $\alpha$-continuous mapping.

## Theorem 44

If $f : X \to Y$ is a fuzzy soft preopen mapping, then, for each soft set $(B,K)$ in Y, $f^{-1}\left(int\big(cl(B,K)\big)\right) \tilde{\subseteq} cl\left(f^{-1}\big((B,K)\big)\right)$.

Proof

It follows from corollary 42.

## Theorem 45

If $f : X \to Y$ is fuzzy soft $\alpha$-continuous and soft preopen, then the inverse image of each fuzzy soft $\alpha$-set is a fuzzy soft $\alpha$-open set.

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Proof

Let $(B, K)$ be any fuzzy soft $\alpha$-open set in Y.

Then

$$f^{-1}((B,K)) \tilde{\sqsubseteq} f^{-1}\left(int\left(cl\left(int((B,K))\right)\right)\right)$$

$$\tilde{\sqsubseteq} int\left(cl\left(int\left(f^{-1}\left(int\left(cl\left(int((B,K))\right)\right)\right)\right)\right)\right) \tilde{\sqsubseteq} int\left(cl\left(int\left(f^{-1}\left(cl\left(int(B,K)\right)\right)\right)\right)\right)$$

By theorem 44 we have $f^{-1}((B,E)) \tilde{\sqsubseteq} int\left(cl\left(f^{-1}\left(int((B,E))\right)\right)\right)$

Since $f$ is a fuzzy soft $\alpha$-continuous mapping, by Theorem 22(5),

$$f^{-1}((B,K)) \tilde{\sqsubseteq} f^{-1}\left(int\left(cl\left(int((B,K))\right)\right)\right)$$

Hence $f^{-1}((B,K))$ is a fuzzy soft $\alpha$-open set.

## CONCLUSION

In this fuzzy soft $\alpha$-open set extended to fuzzy soft $e$-open set and in future, We introduce some new concepts in fuzzy soft topological spaces such as fuzzy soft $\alpha$- open sets, soft $\alpha$-closed sets and soft $\alpha$-continuous functions. We also study relationship between fuzzy soft continuity [6], fuzzy soft semi-continuity [7], and fuzzy soft $\alpha$-continuity of functions defined on fuzzy soft topological spaces. With the help of counterexamples, We show the non-coincidence of these various types of mappings.

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