FUZZY SOFT ALPHA-OPEN SETS AND FUZZY
SOFT ALPHA-CONTINUOUS FUNCTIONS

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INTRODUCTION

In real life situation, the problems in Economics, Engineering, Social sciences, Medical science etc. do not always involve crisp data. So, we cannot successfully use the traditional classical methods because of various types of uncertainties presented in these problems. To exceed these uncertainties, some kind of theories were given like theory of fuzzy set, intutionistic fuzzy set, rough set, bipolar fuzzy set, i.e. which we can use as mathematical tools for dealings with uncertainties. But, all these theories have their inherent difficulties. The reason for these difficulties Molodtsov [1] initiated the concept of soft set theory as a new mathematical tool for dealing with uncertainties which is free from the above difficulties. In [1], Molodtsov successfully applied the soft set theory in several directions, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, Probability, theory of measurement, and so on. After presentation of the operations of soft sets [8], the properties and applications of soft set theory have been studied increasingly [7, 2]. Xiao et al [4] and Pei and Miao [5] discussed the relationship between soft sets and information systems. They showed that soft sets are a class of special information systems. In recent years, many interesting applications of soft set theory have been expanded by embedding the ideas of fuzzy sets [2, 8, 9]. To develop soft set theory, the operations of the soft sets are redefined and a uni-intdecision making method was constructed by using these new operations [11].

Recently, 2011 Shabir and Naz[3] initiated the study of soft topological spaces. They defined soft topology on the collection τ of soft sets over X. Consequently, they defined basic notions of soft topological spaces such as open soft and closed soft sets, soft subspace, soft closure, soft neighborhood of a point, Soft separation axioms, Soft regular spaces and soft normal spaces and established their several properties. Min investigate some properties of these soft separation axioms. In [6], Kandil et al. introduced the notion of soft semi separation axioms. In particular they study the properties of the soft semi regular spaces and soft semi normal spaces. The notion of soft ideal was initiated for the first time by Kandil et al[6]. They also introduced the concept of soft local function. These concepts are discussed with a view to find new soft topologies from the original one, called soft topological spaces with soft ideal (X, τ, E, ʃ). Applications to various fields were further investigated by Kandil et al. The notion of b-open soft sets was initiated for the first time by El-Sheikh and Abd El-latif [12]. Maji et al [9] initiated the study involving bith fuzzy sets and soft sets.

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In [8] the notion of fuzzy soft set was introduced as a fuzzy generalization of soft sets and some basic properties of fuzzy soft sets are discussed in detail. Then, many Scientists such X. Yang et al. improved the concept of fuzziness of soft sets in karl et al. defined the notion of a mapping on classes of fuzzy soft sets, which is fundamental important in fuzzy soft set theory, to improve this work and they studied properties of fuzzy soft images and fuzzy soft inverse images of fuzzy soft sets.

Chang introduced the concept of fuzzy topology on a set X by axiomatizing a collection $\mathcal{X}$ of fuzzy subset of X. Danay et al introduce the definition of fuzzy soft topology over a subset of the initial universe set while Roy and Samanta gave the definition of fuzzy soft topology over the initial universe set. Some fuzzy soft topological properties based on fuzzy pre (resp. semi, $\beta$ -) open soft sets, were introduced in [1, 3].

In the present paper, We introduce some new concepts in fuzzy soft topological spaces such as fuzzy soft $\alpha$- open sets, soft $\alpha$-closed sets and soft $\alpha$-continuous functions. We also study relationship between fuzzy soft continuity [6], fuzzy soft semi-continuity [7], and fuzzy soft $\alpha$-continuity of functions defined on fuzzy soft topological spaces. With the help of counter examples, We show the non-coincidence of these various types of mappings.

PRELIMINARIES

Definition 1 (see [1])

Let $X$ be an initial universe and let $E$ be a set of parameters. Let $P(X)$ denote the power set of $X$ and let $A$ be a nonempty subset of $E$. A pair $(F, A)$ is called a fuzzy soft set over $X$, where $F$ is a mapping given by $F : A \rightarrow P(X)$. In other words, a fuzzy soft set over $X$ is a parameterized family of subsets of the universe $X$. For $e \in A$, $F(e)$ may be considered as the set of $\varepsilon$—approximate elements of the fuzzy soft set $(F, A)$.

Definition 2 (see [8])

A soft set $(F, A)$ over $X$ is called a fuzzy null soft set, denoted by $\varnothing$; if $e \in A, F(e) = \varnothing$

Definition 3 (see [8])

A soft set $(F, A)$ over $X$ is called an fuzzy absolute soft set, denoted by $\bar{A}$; if $e \in A, F(e) = X$.

Definition 4 (see [8])

The union of two fuzzy soft sets of $(F, A)$ and $(G, B)$ over the common universe $X$ is the fuzzy soft set $(H, C)$, where $C = A \sqcup B$ and , for all $e \in C, H(e) =$

$\begin{cases} F(e), & \text{if } e \in A - B, \\ G(e), & \text{if } e \in B - A \\ F(e) \sqcap G(e), & \text{if } e \in A \cap B \end{cases}$

We write $(F, A) \sqcup (G, B) = (H, C)$.

Definition 5 (see [8])

The intersection $(H, C)$ of two fuzzy soft sets $(F, A)$ and $(G, B)$ over a common universe $X$, denoted $(F, A) \cap (G, B)$, is defined as $C = A \cap B$ and $H(e) = F(e) \cap G(e)$ for all $e \in C$.

Definition 6 (see [8])

Let $(F, A)$ and $(G, B)$ be two fuzzy soft sets over a common universe $X$. $(F, A) \subseteq (G, B)$, if $A \subseteq B$, and $H(e) = F(e) \subseteq G(e)$, for all $e \in A$.

Definition 7 (see [2])

Let $\tau$ be the collection of fuzzy soft sets over $X$; then $\tau$ is said to be a fuzzy soft topology on $X$ if it satisfies the following axioms:

1) $\varnothing$ and $\bar{X}$ belong to $\tau$,
2) The union of any number of fuzzy soft sets in $\tau$ belongs to $\tau$,
3) The intersection of any two fuzzy soft sets in $\tau$ belongs to $\tau$.

The triplet $(X, \tau, E)$ is called a fuzzy soft topological space over $X$. Let $(X, \tau, E)$ be a fuzzy soft topological space over $X$; then the members
FUZZY SOFT $\alpha$-OPEN SETS

In this section we introduce fuzzy soft $\alpha$-open set, properties of fuzzy soft $\alpha$-open set and relation between them.

**Definition 9**

A fuzzy soft set $(A, E)$ of a fuzzy soft topological space $(X, \tau, E)$ is called fuzzy soft $\alpha$-open set if

$$\alpha \in \text{int}(\text{cl}(\text{int}(A, E)))$$

The complement of fuzzy soft $\alpha$-open set is called fuzzy soft $\alpha$-closed set.

**Definition 10**

A fuzzy soft set $(A, E)$ is called fuzzy soft preopen set [9] (resp., fuzzy soft semi-open [5]) in a fuzzy soft topological space $X$ if $(A, E) \subseteq \text{int}(\text{cl}(\text{int}(A, E)))$

We will denote the family of all fuzzy soft $\alpha$-open set (resp., $\alpha$-closed set and fuzzy soft preopen set) of a fuzzy soft topological space $(X, \tau, E)$ by $S_{\alpha OS}(X, \tau, E)$ (resp., $S_{\alpha CS}(X, \tau, E)$ and $SPO(X, \tau, E)$).

**Proposition 11**

(1) Arbitrary union of fuzzy soft $\alpha$-open sets is a fuzzy soft $\alpha$-open sets.

(2) Arbitrary intersection of fuzzy soft $\alpha$-closed sets is a fuzzy soft $\alpha$-closed sets.

**Proof**

(1) Let $\{A_i, E : i \in \Lambda\}$ be a collection of fuzzy soft $\alpha$-open sets. Then, for each $i \in \Lambda$,
(A_i,E) \in \text{int} \left( \text{cl} \left( \text{int}(A_i,E) \right) \right). \text{ Now}

\Box (A_i,E) \subseteq \bigcap \text{int} \left( \text{cl} \left( \text{int}(A_i,E) \right) \right)

\subseteq \text{int} \bigcup \text{cl} \left( \text{int}(A_i,E) \right)

= \text{int} \left( \text{cl} \left( \bigcup \text{int}(A_i,E) \right) \right)

\subseteq \text{int} \left( \text{cl} \left( \bigcup (A_i,E) \right) \right).

Hence \Box (A_i,E) \text{ is a fuzzy soft } \alpha \text{-open set.}

(2) Follows immediately from (1) by taking complements.

**Remark 12**

It is obvious that every soft open (resp., fuzzy soft closed) set is a fuzzy soft \alpha-open set (resp., fuzzy soft \alpha-closed set). Similarly, every fuzzy soft \alpha-open set is fuzzy soft semiopen and fuzzy soft pre-open. Thus we have implications as shown in figure 1.

The examples given below show that the converses of these implications are not true.

**Example 13**

Let \( X = \{a, b, c\}, D = \{e_1, e_2, e_3\} \).

\( A, B, C \subseteq D \) where \( A = \{e_1, e_2\}, B = \{e_2, e_3\}, C = \{e_1, e_3\} \) and Let \( \mathcal{X} = \{1_D, \overline{D}, f_{1A}, f_{2B}, f_{3C}\} \), where \( f_{1A}, f_{2B}, f_{3C} \) are fuzzy soft sets over \( X \) defined as follows:

\[
\begin{align*}
\mu_{f_{1A}}^{e_1} &= \{a_{0.2}, b_{0.8}, c_{0.5}\}, \\
\mu_{f_{1A}}^{e_2} &= \{a_{0.7}, b_{0.2}, c_{0.7}\}, \\
\mu_{f_{2B}}^{e_1} &= \{a_{0.8}, b_{0.1}, c_{1}\}, \\
\mu_{f_{2B}}^{e_2} &= \{a_{0.6}, b_{0.1}, c_{0.1}\}, \\
\mu_{f_{3C}}^{e_1} &= \{a_{0.7}, b_{0.5}, c_{0.2}\}.
\end{align*}
\]

Then, \( \mathcal{X} \) defines a fuzzy soft topology on \( X \). Then the fuzzy soft set \( k_D \) where

\[
\begin{align*}
\mu_{k_D}^{e_1} &= \{a_{0.2}, b_{0.6}, c_{0.5}\}, \\
\mu_{k_D}^{e_2} &= \{a_{0.7}, b_{0.1}, c_{0.9}\},
\end{align*}
\]

\( k_D = \{a_{0.6}, b_{0.5}, c_{0.1}\} \), is a fuzzy \( \alpha \)-open set of \( (X, \mathcal{X}, E) \), but is not a fuzzy soft open set.

**Definition 14**

Let \( (X, \tau, E) \) be a fuzzy soft topological space and let \( (A, E) \) be a fuzzy soft set over \( X \).

(1) Fuzzy soft \( \alpha \)-closure of a soft set \( (A, E) \) in \( X \) is denoted by \( S_\alpha \text{cl} \left( (A, E) \right) = \bigcap \{ (F, E) : (F, E) \text{ is a fuzzy soft } \alpha \text{-closed set and } (A, E) \subseteq (F, E) \} \).

(2) Fuzzy soft \( \alpha \)-interior of a fuzzy soft set \( (A, E) \) in \( X \) is denoted by \( S_\alpha \text{int} \left( (A, E) \right) = \bigcup \{ (O, E) : (O, E) \text{ is a fuzzy soft } \alpha \text{-open set and } (A, E) \subseteq (O, E) \} \).

Clearly \( S_\alpha \text{cl} \left( (A, E) \right) \) is the smallest fuzzy soft \( \alpha \)-closed set over \( X \) which contains \( (A, E) \) and \( S_\alpha \text{int} \left( (A, E) \right) \) is the largest fuzzy soft \( \alpha \)-open set over \( X \) which is contained in \( (A, E) \).

**Proposition 15**

Let \( (X, \tau, E) \) be a fuzzy soft topological space and let \( (A, E) \) be a fuzzy soft set over \( X \); then

\[
\begin{align*}
\text{(1) } (A, E) &\in S_\alpha \text{CS} \left( X, \tau, E \right) \iff (A, E) = S_\alpha \text{cl} \left( (A, E) \right); \\
\text{(2) } (A, E) &\in S_\alpha \text{OS} \left( X, \tau, E \right) \iff (A, E) = S_\alpha \text{int} \left( (A, E) \right).
\end{align*}
\]

**Proof**

(1) Let \( (A, E) = S_\alpha \text{cl} \left( (A, E) \right) = \bigcap \left\{ (F, E) : (F, E) \text{ is a fuzzy soft } \alpha \text{-closed set and } (A, E) \subseteq (F, E) \right\} \).

This shows that \( (A, E) \in \bigcap \left\{ (F, E) : (F, E) \text{ is a fuzzy soft } \alpha \text{-closed set and } (A, E) \subseteq (F, E) \right\} \).
Hence $(A, E)$ is fuzzy soft $\alpha$-closed.

Conversely, let $(A, E)$ be fuzzy soft $\alpha$-closed set. Since $(A, E) \in (A, E)$ and $(A, E)$ is a fuzzy soft $\alpha$-closed,

$(A, E) \in \{ (F, E) : (F, E) \text{ is a fuzzy soft } \alpha \text{-closed set and } (A, E) \in (F, E) \}.

Further, $(A, E) \in (F, E)$ for all such $(F, E)$'s.

$(A, E) = \bar{A}

\{ (F, E) : (F, E) \text{ is a fuzzy soft } \alpha \text{-closed set and } (A, E) \in (F, E) \}.

(2) similar to (1).

**Theorem 16**

Let $(X, \tau, E)$ be a fuzzy soft topological space and let $(G, E)$ and $(K, E)$ be two fuzzy soft sets over $X$; then

$1. S_{\alpha}cl\left((G, E)\right)^c = S_{\alpha}int\left((G, E)\right)^c;$

$2. S_{\alpha}int\left((G, E)\right)^c = S_{\alpha}cl\left((G, E)\right)^c;$

$3. (G, E) \in (K, E) \Rightarrow S_{\alpha}int(G, E) \subseteq S_{\alpha}int(K, E);$

$4. S_{\alpha}cl(\Phi) = \Phi \text{ and } S_{\alpha}cl(\bar{X}) = \bar{X};$

$5. S_{\alpha}int(\Phi) = \Phi \text{ and } S_{\alpha}int(\bar{X}) = \bar{X};$

$6. S_{\alpha}cl\left((G, E) \cup (K, E)\right) = S_{\alpha}cl(G, E) \cup S_{\alpha}cl(K, E);$

$7. S_{\alpha}int\left((G, E) \cap (K, E)\right) = S_{\alpha}int(G, E) \cap S_{\alpha}int(K, E);$

$8. S_{\alpha}cl\left((G, E) \cap (K, E)\right) \subseteq S_{\alpha}cl(G, E) \cap S_{\alpha}cl(K, E);$

$9. S_{\alpha}int\left((G, E) \cup (K, E)\right) \subseteq S_{\alpha}int(G, E) \cup S_{\alpha}int(K, E);$

$10. S_{\alpha}int\left(S_{\alpha}cl((G, E))\right) = S_{\alpha}cl((G, E));$

$11. S_{\alpha}int\left(S_{\alpha}int((G, E))\right) = S_{\alpha}int((G, E)).$

**Proof**

Let $(G, E)$ and $(K, E)$ be two fuzzy soft sets over $X$.

$(1) \left(S_{\alpha}cl((G, E))\right)^c = (\bar{G}, E) \subseteq (\bar{K}, E)$

$(\bar{G}, E) \subseteq (\bar{K}, E) \Rightarrow S_{\alpha}int(G, E) \subseteq S_{\alpha}int(K, E);$

$(2)$ Similar to $(1)$.

$(3)$ It follows from Definition 15.

$(4)$ Since $\Phi$ and $\bar{X}$ are fuzzy soft $\alpha$-closed sets so, $S_{\alpha}cl(\Phi) = \Phi$ and $S_{\alpha}cl(\bar{X}) = \bar{X}.$

$(5)$ Since $\Phi$ and $\bar{X}$ are fuzzy soft $\alpha$-open sets so, $S_{\alpha}int(\Phi) = \Phi$ and $S_{\alpha}int(\bar{X}) = \bar{X}.$

$(6)$ We have $(G, E) \in ((G, E) \cup (K, E))$ and $(K, E) \in ((G, E) \cup (K, E)).$

Then by proposition $S_{\alpha}cl(G, E) \in S_{\alpha}cl((G, E) \cup (K, E))$ and $S_{\alpha}cl(K, E) \in S_{\alpha}cl((G, E) \cup (K, E)) \Rightarrow S_{\alpha}cl(K, E) \subseteq S_{\alpha}cl(G, E) \subseteq \bar{X}$

Now, $S_{\alpha}cl(G, E), S_{\alpha}cl(K, E) \subseteq S_{\alpha}CS(X, \tau, E) \Rightarrow S_{\alpha}cl(G, E) \subseteq S_{\alpha}cl(K, E) \subseteq S_{\alpha}CS(X, \tau, E).$

Then $(G, E) \subseteq S_{\alpha}cl(G, E) \quad S_{\alpha}cl(K, E) \subseteq S_{\alpha}cl(K, E)$ imply $(G, E) \cup (K, E) \subseteq S_{\alpha}cl(G, E) \cup S_{\alpha}cl(K, E).$ That
is, $S_\alpha \text{cl}(G, E) \sqcup S_\alpha \text{cl}(K, E)$ is a fuzzy soft $\alpha$-closed set containing $(G, E) \sqcup (K, E)$.

But $S_\alpha \text{cl}((A, E) \sqcup (K, E))$ is the smallest fuzzy soft $\alpha$-closed set containing $(G, E) \sqcup (K, E)$.

Hence $S_\alpha \text{cl}((G, E) \sqcup (K, E)) \sqsubseteq S_\alpha \text{cl}(G, E)$.

(7) Similar to (6).

(8) We have $((G, E) \sqcap (K, E)) \sqsubseteq (G, E)$ and $((G, E) \sqcap (K, E)) \sqsubseteq (K, E) \Rightarrow S_\alpha \text{cl}([(G, E) \sqcap (K, E)]) \sqsubseteq S_\alpha \text{cl}(G, E)$ and $S_\alpha \text{cl}((G, E) \sqcap (K, E)) \sqsubseteq S_\alpha \text{cl}(K, E) \Rightarrow S_\alpha \text{cl}((G, E) \sqcap (K, E)) \sqsubseteq S_\alpha \text{cl}(G, E) \sqcap S_\alpha \text{cl}(K, E)$.

(9) Similar to (8).

(10) Since $(S_\alpha \text{cl}(G, E)) \in S_\alpha \text{CS}(X, \tau, E)$ so by proposition 16(1), $S_\alpha \text{cl}([S_\alpha \text{cl}(G, E)]) = (S_\alpha \text{cl}(G, E))$.

(11) Since $(S_\alpha \text{int}(G, E)) \in S_\alpha \text{OS}(X, \tau, E)$ so by proposition 16(2), $S_\alpha \text{int}([S_\alpha \text{int}(G, E)]) = (S_\alpha \text{int}(G, E))$.

**Theorem 17**

If $(G, E)$ is any fuzzy soft set in a fuzzy soft topological space $(X, \tau, E)$, then following are equivalent:

1. $(G, E)$ is fuzzy soft $\alpha$-closed set;
2. $\text{int}(\text{cl}(\text{int}(G, E)^c)) \sqsubseteq (G, E)^c$;
3. $\text{cl}(\text{int}(G, E)) \sqsubseteq (G, E)$;
4. $(G, E)$ is fuzzy soft $\alpha$-open set.

**Proof**

(1) $\Rightarrow$ (2) If $(G, E)$ is fuzzy soft $\alpha$-closed set, then

$$\text{cl}(\text{int}(\text{cl}(G, E))) \subseteq (G, E)$$

$$\text{cl}(\text{int}(G, E)^c) \subseteq (G, E)^c$$

(2) $\Rightarrow$ (3)

$$\text{int}(\text{cl}(\text{int}(G, E)^c)) \subseteq ((G, E)^c)^c$$

$$\Rightarrow \text{cl}(\text{int}(G, E)^c) \subseteq (G, E)$$

(3) $\Rightarrow$ (4) It is obvious from Definition 9;

(4) $\Rightarrow$ (1) It is obvious from Definition 9.

**FUZZY SOFT $\alpha$-CONTINUITY**

In this section we introduce the fuzzy soft $\alpha$-continuous, properties & relations

**Definition 18**

Let $(X, E)$ and $(Y, K)$ be fuzzy soft classes. Let $u : X \to Y$ and $p : E \to K$ be mapping $f : (X, E) \to (Y, K)$ defined as follows: for a fuzzy soft set $(F, A)$ in $(X, E)$, $(f(F, A), B), B = p(A) \subseteq K$ is a fuzzy soft set in $(Y, K)$ given by

$$f(F, A)(\beta) = u \left( \frac{\text{UF}(\alpha)}{\text{UF}(\beta) \cap A} \right)$$

for $\beta \in K$.

Let $B = K$, then we will write $f(F, A)(K)$ as $f(F, A)$.

**Definition 19**

Let $f : (X, E) \to (Y, K)$ be a mapping from a fuzzy soft class $(X, E)$ to another fuzzy soft class $(Y, K)$ and $(G, C)$ of a fuzzy soft set in fuzzy soft class $(Y, K)$, where $C \subseteq K$. Let $u : X \to Y$ and $p : E \to K$ be mappings. Then $(f^{-1}(G, C), D), D = p^{-1}(C)$, is a fuzzy soft set in fuzzy soft classes $(X, E)$, defined as $f^{-1}(G, C)(\alpha) = u^{-1}(G(p(\alpha)))$ for $\alpha \in D \subseteq E$. $(f^{-1}(G, C), D)$ is called a fuzzy soft inverse image of $(G, C)$. Hereafter we will write $(f^{-1}(G, C), E)$ as $f^{-1}(G, C)$. © Eureka Journals 2019. All Rights Reserved. ISSN: 2581-7620
Theorem 20

Let \( f : (X, E) \rightarrow (Y, K) \), \( u : X \rightarrow Y \) and \( p : E \rightarrow K \) be mappings. Then for fuzzy soft sets \((F_{i}, A_{i})\) and \((G, B)\) and a family of fuzzy soft sets \((F_{i}, A_{i})\) in the fuzzy soft class \((X, E)\), one has:

1. \( f(\emptyset) = \emptyset \),
2. \( f(\bar{X}) = \bar{Y} \),
3. \( f((F, A) \sqcup (G, B)) = f(F, A) \sqcup f(G, B) \) in general \( f\left(\bigcup_{i} (F_{i}, A_{i})\right) = \bigcup_{i} f(F_{i}, A_{i}) \),
4. \( f((F, A) \sqcup (G, B)) \sqcap f((F, A) \sqcap f(G, B)) \) in general \( f\left(\bigcap_{i} (F_{i}, A_{i})\right) = \bigcap_{i} f(F_{i}, A_{i}) \),
5. \( f^{-1}(\phi) = \emptyset \),
6. \( f^{-1}(Y) = \bar{X} \),
7. \( f^{-1}(F, A) \sqcap (G, B) = f^{-1}(F, A) \sqcap f^{-1}(G, B) \) in general \( f^{-1}\left(\bigcup_{i} (F_{i}, A_{i})\right) = \bigcup_{i} f^{-1}(F_{i}, A_{i}) \),
8. \( f^{-1}((F, A) \sqcup (G, B)) = f^{-1}(F, A) \sqcup f^{-1}(G, B) \) in general \( f^{-1}\left(\bigcap_{i} (F_{i}, A_{i})\right) = \bigcap_{i} f^{-1}(F_{i}, A_{i}) \),
9. \( f^{-1}(F, A) \sqcap (G, B) = f^{-1}(F, A) \sqcap f^{-1}(G, B) \) in general \( f^{-1}\left(\bigcup_{i} (F_{i}, A_{i})\right) = \bigcup_{i} f^{-1}(F_{i}, A_{i}) \),
10. If \((F, A) \sqsubseteq (G, B)\), then \( f^{-1}(F, A) \sqsubseteq f^{-1}(G, B) \).

Definition 21

A mapping \( f : (X, \tau, E) \rightarrow (Y, v, K) \) is said to be fuzzy soft mapping if \((X, \tau, E)\) and \((Y, v, K)\) are fuzzy soft topological spaces and \( u : X \rightarrow Y \) and \( p : E \rightarrow K \) are mappings.

Throughout the paper, the spaces \( X \) and \( Y \) \((\text{or} \ (X, \tau, E) \text{ and } (Y, v, K))\) stand for fuzzy soft topological space assumed unless otherwise stated.

Definition 22

A soft mapping \( f : X \rightarrow Y \) is said to be fuzzy soft \( \alpha \)-continuous if the inverse image of each fuzzy soft \( \alpha \)-open subset of \( Y \) is a fuzzy soft \( \alpha \)-open set in \( X \).

Example 23

Let \( X = \{x_{1}, x_{2}, x_{3}\}, Y = \{y_{1}, y_{2}, y_{3}\}, E = \{e_{1}, e, e_{2}\}, K = \{k_{1}, k_{2}, k_{3}\}, \tau = \{\emptyset, X, (F, E), (F, E) = \{(e_{1}, \{x_{1}\}), (e_{2}, \{x_{2}\}), (e_{3}, \{x_{1}, x_{3}\})\} \) and let \((X, \tau, E)\) and \((Y, v, K)\) be fuzzy soft topological spaces.

Define \( u : X \rightarrow Y \) and \( p : E \rightarrow K \) as

\[
\begin{align*}
u(x_{1}) & = \{y_{1}\}, u(x_{2}) = \{y_{3}\}, u(x_{3}) = \{y_{2}\}, \\
p(e_{1}) & = \{y_{1}\}, p(e_{2}) = \{k_{3}\}, p(e_{3}) = \{k_{1}\}
\end{align*}
\]

Let \( f_{pu} : (X, \tau, E) \rightarrow (Y, v, K) \) be a soft mapping. Then \((G, K)\) is a fuzzy soft open in \( Y \) and \( f_{pu}^{-1}(G, K) = (F, E) \) is a fuzzy soft \( \alpha \)-open in \( X \).

Therefore, \( f_{pu} \) is a fuzzy soft \( \alpha \)-continuous function.

Theorem 24

Let \( f : X \rightarrow Y \) be a mapping from a fuzzy soft space \( X \) to soft space \( Y \). Then the following statements are true:

1. \( f \) is fuzzy soft \( \alpha \)-continuous;
2. for each fuzzy soft singleton \((P, E)\) in \( X \) and each fuzzy soft open set \((O, K)\) in \( Y \), there is a fuzzy soft \( \alpha \)-open set \((U, E)\) in \( X \) such that \((P, E) \sqsubseteq (U, E)\) and \( f((U, E)) = (O, K)\);
3. The inverse image of each fuzzy soft closed set in \( Y \) is fuzzy soft \( \alpha \)-closed in \( X \);
4. \( f\left(\text{cl}\left(\text{int}(cl(A, E))\right)\right) \sqsubseteq \text{cl}(f(A, E)) \), for each fuzzy soft set \((A, E)\) in \( X \);
5. \( \text{cl}\left(\text{int}\left(\text{cl}(f^{-1}(B, K))\right)\right) \sqsubseteq f^{-1}(\text{cl}(B, K)) \), for each fuzzy soft set \((B, K)\) in \( Y \).
Proof

(1) ⇒ (2) Since \((O, K)\) is fuzzy soft open in \(Y\) and \(\exists (P, E) \subseteq (O, K)\), so \((P, E) \subseteq f^{-1}(O, K)\) and \(f^{-1}(O, K)\) is a fuzzy soft \(\alpha\)-open set in \(X\). Put \((U, E) = f^{-1}(O, K)\). Then \((P, E) \subseteq (U, E)\) and \(f((U, E)) \subseteq (O, K)\).

\((2) \Rightarrow (1)\) Let \((O, K)\) be a fuzzy soft open set in \(Y\) such that \((P, E) \subseteq f^{-1}(O, K)\) and thus there exists \((U, E) \in S_{a}\) such that \((P, E) \subseteq (U, E)\) and \(f((U, E)) \subseteq (O, K)\). Then \((P, E) \subseteq (U, E) \subseteq f^{-1}(O, K) = \Box (U, E) \in S_{a}(X)\). Hence \(f^{-1}(O, K) \in S_{a}(X)\) and there is a fuzzy soft \(\alpha\)-set in \(X\).

\((1) \Rightarrow (3)\) Let \((G, K)\) be a fuzzy soft closed set in \(Y\). Then \((G, K)\) is fuzzy soft open in \(Y\). Thus \(f^{-1}(G, K) \in S_{a}(X)\). Hence \(f^{-1}(G, K)\) is a fuzzy soft \(\alpha\)-set in \(X\).

\((3) \Rightarrow (4)\) Let \((S, K)\) be a fuzzy soft set in \(Y\). Then \(cl(f(S, K))\) is a fuzzy soft closed set in \(Y\), so that \(f^{-1}(cl(S, K))\) is fuzzy soft \(\alpha\)-closed in \(X\).

Therefore, we have \(f^{-1}(cl(S, K)) \supseteq cl\left(int\left(cl\left(f^{-1}(cl(S, K))\right)\right)\right)\)

\(\supseteq cl\left(int\left(cl(S, K))\right)\right) = cl\left(int(S, K)\right)\).

\((4) \Rightarrow (5)\) Since \((B, K)\) is a fuzzy soft set in \(Y\), then \(f^{-1}(B, K)\) is a fuzzy soft set in \(X\); thus by hypothesis we have \(cl\left(int\left(cl\left(f^{-1}(B, K))\right)\right)\right) \supseteq cl\left(f^{-1}(B, K)\right)\).

\(\supseteq cl(B, K)\) and that is, \(cl\left(int\left(cl\left(f^{-1}(B, K))\right)\right)\right) \supseteq cl^{-1}(cl(B, K))\).

\((5) \Rightarrow (1)\) Let \((O, K)\) be a soft open in \(Y\). Let \((U, K) = (O, K)^{c}\) and \((D, E) = f^{-1}(U, K))\). By \(cl\left(int\left(cl\left(f^{-1}(U, K))\right)\right)\right) \supseteq cl\left(int\left(cl\left((O, K)^{c})\right)\right)\right) \supseteq f^{-1}(O, K)\).

Corollary 25

Let \(f : X \rightarrow Y\) be a fuzzy soft \(\alpha\)-continuous mapping. Then

\((1) f(cl(A, E)) \subseteq cl(f(cl(A, E)))\), for each \((A, E) \in SPO(X)\);

\((2) cl(f^{-1}(B, K)) \subseteq f^{-1}(cl(B, K))\), for each \((B, K) \in SPO(Y)\).

Proof

Since for each \((A, E) \in SPO(X)\), \(cl((A, E)) = cl\left(int\left(cl((A, E))\right)\right)\), therefore the proof follows directly from statements (4) and (5) of theorem 26.

Definition 26

A Fuzzy Soft mapping \(f : X \rightarrow Y\) is called fuzzy soft pre-continuous (resp., fuzzy soft semicontinuous [7]) if the inverse image of each fuzzy soft open set in \(Y\) is fuzzy soft preopen (resp., fuzzy soft semiopen) in \(X\).

Remark 27

It is clear that every fuzzy soft \(\alpha\)-continuous map is fuzzy soft semicontinuous and fuzzy soft precontinuous. Every fuzzy soft continuous map
is fuzzy soft $\alpha$-continuous. Thus we have implications as shown in fig 2.

The converses of these implications are not true, which is clear from the following examples.

**Example 28**

Let $X = \{x_1, x_2, x_3, x_4\}, Y = \{y_1, y_2, y_3, y_4\}, E = \{e_1, e_2, e_3\}, K = \{k_1, k_2, k_3\}$ and $(X, \tau, E)$ and Let $(Y, u, K)$ be fuzzy soft topological spaces.

Fuzzy Soft continuity

Fuzzy Soft semicontinuity

Fuzzy Soft $\alpha$-continuity

Fuzzy Soft precontinuity

Define $u: X \to Y$ and $p: E \to K$ as

$u(x_1) = \{y_2\}, u(x_2) = \{y_4\}, u(x_3) = \{y_1\}, u(x_4) = \{y_3\}$,

$p(e_1) = \{k_2\}, p(e_2) = \{k_1\}, p(e_3) = \{k_3\}$.

Let us consider the fuzzy soft topology $\tau$ given in example 14; that is, $\tau = \{\Phi, X, (F_1, E), (F_2, E), (F_3, E), \ldots, (F_{15}, E)\}$,

$v = \{\Phi, Y, (F, K)\}$ and $(F, K) = \{(k_1, \{y_1, y_3, y_4\}), (k_2, \{y_1, y_2, y_4\}), (k_3, \{y_2, y_4\})\}$ and let mapping $f_{pu}: (X, \tau, E) \to (Y, u, K)$ be a fuzzy soft mapping. Then $(F, K)$ is a fuzzy soft open in $Y$ and $f_{pu}^{-1}((F, K)) = \{(e_1, \{x_1, x_2, x_3\}), (e_2, \{x_2, x_3, x_4\}), (e_3, \{x_1, x_2, x_3\})\}$ is a fuzzy soft $\alpha$-open but not fuzzy soft open in $X$.

Therefore, $f_{pu}$ is a fuzzy soft $\alpha$-continuous function but not fuzzy soft continuous function.

Therefore, $f_{pu}$ is a fuzzy soft $\alpha$-continuous function but not fuzzy soft continuous function.

**Example 29**

Let $X = \{a, b, c\}, Y = \{x, y\}, E = \{e_1, e_2\}$ and $K = \{k_1, k_2\}$. Define

$A \subseteq E, B \subseteq K, f_A =$

\[
\{(e_1, \{0.5, 0.5, 0.3\}), (e_2, \{0.4, 0.4, 0.2\})\}
\]

$\in (X, E)$ and

$g_B = \{(k_1, \{0.5, 0.3\}), (k_2, \{0.4, 0.2\})\} \in (Y, K)$

Define fuzzy soft topologies $\tau_E: E \to (X, E)$ and $\tau_B: K \to (Y, K)$ as follows:

$$
\tau_E(h_G) = \begin{cases}
1, & \text{if } h_G = \varphi, \tilde{E} \\
\frac{1}{2}, & \text{if } h_G = f_A, E^{\tilde{0}4} \\
\frac{2}{3}, & \text{if } h_G = f_A \cup E^{\tilde{0}4} \\
0, & \text{Otherwise}
\end{cases}
$$

$$
\tau_B(w_D) = \begin{cases}
1, & \text{if } w_D = \varphi, \tilde{E} \\
\frac{1}{2}, & \text{if } w_D = g_B \\
0, & \text{Otherwise}
\end{cases}
$$

Consider the maps $\varphi: X \to Y$ and $\psi: E \to K$ defined by $\varphi(a) = \varphi(b) = X, \varphi(c) = y, \psi_1 = k_1$ and $\psi_2 = k_2$

Therefore, for each $e \in E, k \in K$ and $r \in \mathcal{I}_0$ define the fuzzy soft operators $\alpha = E \times (X, E) \times \mathcal{I}_0 \to (X, E)$ as follows:

$$
\alpha(e, w_D, r) = w_D.
$$

Then the map $\varphi: (X, E) \to (Y, \tau)\ast$ is fuzzy soft continuous.

**Example 30**

Let $X = Y = \{a, b, c\}, E = \{e_1, e_2, e_3\}$ and $A \subseteq E$ where $A = \{e_1, e_2\}$. Let $f_{pu}: (X, \mathcal{I}_1, E) \to (Y, \mathcal{I}_2, K)$ be the constant soft mapping where $\mathcal{I}_1$ is the indiscrete fuzzy soft topology and $\mathcal{I}_2$ is the discrete fuzzy soft topology such that $u(x) = a \forall x \in X$ and $p(e) = e_1 \forall e \in E$. Let $f_A$ be fuzzy soft set over $Y$ defined as follows:

$\mu_{f_A}^\epsilon = \{a_{0.1}, b_{0.5}, c_{0.6}\}$

$\mu_{f_A}^\delta = \{a_{0.6}, b_{0.2}, c_{0.5}\}$
Then $f_A \in \mathcal{X}_2$. Now we find $f_{pu}^{-1}(f_A)$ as follows

$$f_{pu}^{-1}(f_A)(e_1)(a) = f_A(p(e_1))(u(a))$$

$$= f_A(e_1)(a)$$

$$= 0.6$$

$$f_{pu}^{-1}(f_A)(e_1)(b) = f_A(p(e_1))(u(b))$$

$$= f_A(e_1)(a)$$

$$= 0.6$$

$$f_{pu}^{-1}(f_A)(e_1)(c) = f_A(p(e_1))(u(c))$$

$$= f_A(e_1)(a)$$

$$= 0.6$$

$$f_{pu}^{-1}(f_A)(e_2)(a) = f_A(p(e_2))(u(a))$$

$$= f_A(e_1)(a)$$

$$= 0.6$$

$$f_{pu}^{-1}(f_A)(e_2)(b) = f_A(p(e_2))(u(b))$$

$$= f_A(e_1)(a)$$

$$= 0.6$$

$$f_{pu}^{-1}(f_A)(e_2)(c) = f_A(p(e_2))(u(c))$$

$$= f_A(e_1)(a)$$

$$= 0.6$$

$$f_{pu}^{-1}(f_A)(e_3)(a) = f_A(p(e_3))(u(a))$$

$$= f_A(e_1)(a)$$

$$= 0.6$$

$$f_{pu}^{-1}(f_A)(e_3)(b) = f_A(p(e_3))(u(b))$$

$$= f_A(e_1)(a)$$

$$= 0.6$$

$$f_{pu}^{-1}(f_A)(e_3)(c) = f_A(p(e_3))(u(c))$$

$$= f_A(e_1)(a)$$

$$= 0.6$$

Hence $f_{pu}^{-1}(f_A) \not\in F\alpha OS(X)$

Therefore $f_{pu}$ is not fuzzy soft $\alpha$-continuous function. On the other hand, if we consider $\mathcal{X}_1$ is the discrete fuzzy soft topology. In this case $f_{pu}$ will be fuzzy $\alpha$-continuous soft function

**Theorem 31**

Let $(A,E) \in SPO(X)$ and $(B,E) \in \mathcal{X}_2$. Then $(A,E) \cap (B,E) \in \mathcal{X}_2$. Therefore $(A,E)$ is a fuzzy soft $\alpha$-open set of $(A,E)$.

**Theorem 32**

If $f : X \to Y$ is a fuzzy soft $\alpha$-continuous mapping and $(A,E) \in SPO(X)$, then $f |_{(A,E)}$ is fuzzy soft $\alpha$-continuous mapping.

Proof
Let \((B, E)\) in \(Y\) be a fuzzy soft open set. Then \(f^{-1}(B, E) \in S_{\alpha}(X)\) and since \((A, E)\) is a fuzzy soft preopen set in \(X\), by theorem 26, we have 

\[
(A, E) \cap f^{-1}(B, E) = f\left((A, E)\right)^{-1}((B, E)) \in S_{\alpha}SO(X)((A, E)).
\]

Therefore \(f\left((A, E)\right)^{-1}((B, E))\) is a fuzzy soft \(\alpha\)-continuous mapping.

**Theorem 33**

A fuzzy soft function \(f : X \rightarrow Y\) is fuzzy soft \(\alpha\)-continuous if and only if

\[
f\left(s_{\alpha}cl((F, E))\right) \subseteq cl(f((F, E)))
\]

for every fuzzy soft set \((F, E)\) of \(X\).

Proof

Let \(f : X \rightarrow Y\) be fuzzy soft \(\alpha\)-continuous. Now \(cl(f((F, E)))\) is a fuzzy soft closed set of \(Y\);

So by fuzzy soft \(\alpha\)-continuity of \(f\),

\[
f^{-1}\left(cl(f((F, E)))\right) \subseteq f^{-1}\left(cl(f((F, E)))\right)
\]

is fuzzy soft \(\alpha\)-closed and

\(F, E) \subseteq f^{-1}\left(cl(f((F, E)))\right)\).

But \(s_{\alpha}cl(F, E)\) is the smallest \(\alpha\)-closed set containing \((F, E)\);

Hence \(s_{\alpha}cl(F, E) \subseteq f^{-1}\left(cl(f((F, E)))\right)\)

\[
\Rightarrow f\left(s_{\alpha}cl(F, E)\right) \subseteq cl(f((F, E)))
\]

Conversely, Let \((F, K)\) be any fuzzy soft closed set of \(Y\)

\[
\Rightarrow f^{-1}((F, K)) \in X\text{ and by hypothesis}
\]

\[
\Rightarrow f\left(s_{\alpha}cl(f^{-1}((F, K)))\right) \subseteq cl\left(f\left(f^{-1}((F, K))\right)\right)
\]

\[
\Rightarrow f\left(s_{\alpha}cl(f^{-1}((F, K)))\right) \subseteq cl(F, K) = (F, K)
\]

⇒ \(s_{\alpha}cl\left(f^{-1}((F, K))\right) = f^{-1}((F, K))\); hence fuzzy soft \(\alpha\)-closed. Consequently, \(f\) fuzzy soft \(\alpha\)-continuous.

**Theorem 34**

A Fuzzy Soft function \(f : X \rightarrow Y\) is fuzzy soft \(\alpha\)-continuous if and only if \(f^{-1}(int(H, K)) \subseteq s_{\alpha}intf^{-1}(int(H, K))\) for every fuzzy soft set \((H, K)\) of \(Y\).

Proof

Let \(f : X \rightarrow Y\) be fuzzy soft \(\alpha\)-continuous. Now for any fuzzy soft set \((G, E)\) in \(X\), int\(f((G, E)))\) is a fuzzy soft open set in \(Y\); since \(f\) is fuzzy soft \(\alpha\)-continuity, then \(f^{-1}\left(int(f((G, E)))\right)\) is fuzzy soft \(\alpha\)-open and \(f^{-1}\left(int(f((G, E)))\right) \subseteq (G, E), As \)

\(s_{\alpha}int(G, E)\) is the largest fuzzy soft \(\alpha\)-open set contained in \((G, E), f^{-1}\left(int(f(G, E)))\right) \subseteq s_{\alpha}int(G, E)\).

Conversely, take a fuzzy soft open set \((G, K)\) in \(Y\). Then \(f^{-1}(int(G, K)) \subseteq s_{\alpha}intf^{-1}(int(G, K)) \Rightarrow f^{-1}(G, K) \subseteq s_{\alpha}intf^{-1}(G, K) \Rightarrow f^{-1}(G, K)\) is fuzzy soft \(\alpha\)-open.

**FUZZY SOFT \(\alpha\)-OPEN AND FUZZY SOFT \(\alpha\)-CLOSED MAPPINGS**

**Definition 35**

A fuzzy soft mapping \(f : X \rightarrow Y\) is called fuzzy soft \(\alpha\)-open (resp., fuzzy soft \(\alpha\)-closed) mapping if the image of each fuzzy soft open (resp., fuzzy soft closed) set in \(X\) is a fuzzy soft \(\alpha\)-open set (resp., fuzzy soft \(\alpha\)-closed set) in \(Y\).

**Definition 36**

A fuzzy soft mapping \(f : X \rightarrow Y\) is called fuzzy soft pre-open (resp., fuzzy soft semiopen[7]) if the image of each fuzzy soft open set in \(X\) is fuzzy soft preopen (resp., fuzzy soft semiopen) in \(Y\).
Clearly a fuzzy soft open map is fuzzy soft α-open map is fuzzy soft preopen as well as fuzzy soft α-open. Similar implications hold for fuzzy soft closed mappings.

**Theorem 37**

A fuzzy soft mapping \( f : X \to Y \) is fuzzy soft α-closed if and only if \( \text{sac}(f(A,E)) \subseteq f(cl(A,E)) \) for each fuzzy soft set \( (A,E) \) in \( X \).

**Proof**

Let \( \text{sac}(f(A,E)) \subseteq f(cl(A,E)) \). By the definition of fuzzy soft α-closure, we have \( f(A,E) = f(cl(A,E)) \) and so \( f(cl(A,E)) \) is fuzzy soft α-closed set and \( f \) is a fuzzy soft α-closed mapping.

Conversely, if \( f \) is fuzzy soft α-closed, then \( f(cl(A,E)) \) is a fuzzy soft α-closed set containing \( f(A,E) \) and therefore \( \text{sac}(f(A,E)) \subseteq f(cl(A,E)) \).

**Theorem 38**

A fuzzy soft function \( f : X \to Y \) is fuzzy soft α-open if and only if \( f(int(F,E)) \subseteq \text{sα int}(f((F,E))) \) for every fuzzy soft set \( (F,A) \) in \( X \).

**Proof**

If \( f : X \to Y \) is fuzzy soft α-open, then \( f(int(F,E)) = \text{sα int}(f((F,E))) \subseteq \text{sα int}(f(int(F,E))) \)

\( \subseteq \text{sα int}(f(F,E)) \).

On the other hand, take a fuzzy soft open set \( (G,E) \) in \( X \). Then by hypothesis, \( f((G,E)) = f(int(G,E)) \subseteq \text{sα int}(f((G,E))) \Rightarrow f((G,E)) \) is fuzzy soft α-open in \( Y \).

**Theorem 39**

Let \( f : X \to Y \) be a fuzzy soft α-open (resp., fuzzy soft α-closed) mapping. If \( (B,K) \) is a fuzzy soft set in \( Y \) and \( (A,E) \) is a fuzzy soft closed (resp., fuzzy soft open) set in \( X \), containing \( f^{-1}((B,K)) \); then there exists a fuzzy soft α-closed (resp., fuzzy soft α-open) set \( (C,K) \) in \( Y \), such that \( (B,K) \subseteq (C,K) \) and \( f^{-1}((C,K)) \subseteq (A,E) \).

**Proof**

Let \( (C,K) = (f(A,E))^c \). Since \( f^{-1}((B,K)) \subseteq (A,E) \), we have \( f((A,E))^c \subseteq (B,K) \). Since \( f \) is fuzzy soft α-open (resp., fuzzy soft α-closed), then \( (C,K) \) is a fuzzy soft α-closed set (resp., fuzzy soft α-open set) if

\[
 f^{-1}((C,K)) = \left( f^{-1}(f((A,E))^c) \right)^c \subseteq (A,E)^c \]

and hence \( (B,K) \subseteq (C,K) \) and \( f^{-1}((C,K)) \subseteq (A,E) \).

**Corollary 40**

If \( f : X \to Y \) is a fuzzy soft α-open mapping, then

1. \( f^{-1} \left( cl \left( \text{int} \left( cl((B,K)) \right) \right) \right) \subseteq cl \left( f^{-1}((B,K)) \right) \), for every fuzzy soft set \( (B,K) \) in \( Y \);

2. \( f^{-1} \left( cl((C,K)) \right) \subseteq cl \left( f^{-1}((C,K)) \right) \), \( (C,K) \in SPO(Y) \).

**Proof**

(1) \( cl \left( f^{-1}((B,K)) \right) \) is a fuzzy soft closed in \( X \), containing \( f^{-1}((B,K)) \), for a fuzzy soft set \( (B,K) \) in \( Y \).

By theorem 44, there exists a fuzzy soft α-closed set \( (F,K) \) in \( Y \), and \( (B,K) \subseteq (F,K) \) such that \( f^{-1}((F,K)) \subseteq cl \left( f^{-1}((B,K)) \right) \).

Thus \( f^{-1} \left( cl \left( \text{int} \left( cl((B,K)) \right) \right) \right) \subseteq \)
\[ \text{Proof} \]

We have \( f^{-1}((B, K)) \subseteq f^{-1}\left( \text{int}\left( \text{cl}\left( (F, E) \right) \right) \right) \)

\[ \subseteq \text{int}\left( \text{cl}\left( f^{-1}\left( \text{int}\left( \text{cl}\left( (B, K) \right) \right) \right) \right) \right) \]

\[ \text{Since } f \text{ is a soft } \alpha \text{-open map, we have, by corollary 42, } \]

\[ f^{-1}\left( (B, K) \right) \subseteq \text{int}\left( \text{cl}\left( f^{-1}\left( \text{int}\left( \text{cl}\left( (B, K) \right) \right) \right) \right) \right) \]

\[ \subseteq \text{cl}\left( \text{int}\left( f^{-1}\left( \text{cl}\left( (B, K) \right) \right) \right) \right) \]

\[ = \text{cl}\left( \text{int}\left( f^{-1}\left( (B, K) \right) \right) \right). \]

Therefore \( f^{-1}\left( (B, K) \right) \) is a fuzzy soft preopen set in \( X \).

**Theorem 42**

If \( f : X \to Y \) is a fuzzy soft precontinuous and fuzzy soft \( \alpha \)-semicontinuous, then \( f \) is fuzzy soft \( \alpha \)-continuous.

**Proof**

We have \( f^{-1}\left( (B, K) \right) \subseteq f^{-1}\left( \text{int}\left( \text{cl}\left( (B, K) \right) \right) \right) \)

\[ \subseteq \text{int}\left( \text{cl}\left( f^{-1}\left( \text{int}\left( \text{cl}\left( (B, K) \right) \right) \right) \right) \right) \]

\[ = \text{int}\left( \text{cl}\left( f^{-1}\left( (B, K) \right) \right) \right). \]

It follows from corollary 42.
Proof
Let \((B,K)\) be any fuzzy soft \(\alpha\)-open set in \(Y\).

\[
\in \text{int}(\text{cl}(\text{int}(f^{-1}(\text{cl}(\text{int}(B,K)))))) \subseteq \text{int}(\text{cl}(f^{-1}(\text{int}(\text{cl}(\text{int}(B,K))))))
\]

By theorem 44 we have

\[
f^{-1}((B,E)) \subseteq \text{int}(\text{cl}(f^{-1}(\text{int}(B,E))))
\]

Since \(f\) is a fuzzy soft \(\alpha\)-continuous mapping, by Theorem 22(5),

\[
f^{-1}((B,K)) \subseteq f^{-1}(\text{int}(\text{cl}(\text{int}(B,K))))
\]

Hence \(f^{-1}((B,K))\) is a fuzzy soft \(\alpha\)-open set.

**CONCLUSION**

In this fuzzy soft \(\alpha\)-open set extended to fuzzy soft \(e\)-open set and in future, We introduce some new concepts in fuzzy soft topological spaces such as fuzzy soft \(\alpha\)-open sets, soft \(\alpha\)-closed sets and soft \(\alpha\)-continuous functions. We also study relationship between fuzzy soft continuity [6], fuzzy soft semi-continuity [7], and fuzzy soft \(\alpha\)-continuity of functions defined on fuzzy soft topological spaces. With the help of counterexamples, We show the non-coincidence of these various types of mappings.

**REFERENCES**


